A Brief History of American K-12 Mathematics Education in the 20th Century

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Published in:

Mathematical Cognition, Edited by James Royer

Introduction

In January 1998, when U.S. Education Secretary Richard Riley called for an end to the "math wars" in a speech before a joint meeting of the American Mathematical Society and the Mathematical Association of America, he could not have known that within two years, the department he directed would become the focus of the very math wars he sought to quell. In October 1999, the U.S. Department of Education recommended to the nation's 15,000 school districts a list of math books, including several that had been sharply criticized by mathematicians and parents of school children across the country for much of the preceding decade. Within a month of that release, 200 university mathematicians added their names to an open letter to Secretary Riley calling upon his department to withdraw those recommendations. The list of signatories included seven Nobel laureates and winners of the Fields Medal, the highest international award in mathematics, as well as math department chairs of many of the top universities in the country, and several state and national education leaders.[1] By the end of the year 1999, the U.S. Secretary of Education had himself become embroiled in the nation's math wars.

Mathematics education policies and programs for U.S. public schools have never been more contentious than they were during the decade of the 1990s. The immediate cause of the math wars of the 90s was the introduction and widespread distribution of new math textbooks with radically diminished content, and a dearth of basic skills. This led to organized parental rebellions and criticisms of the new math curricula by mathematicians and other professionals.

In some respects the education wars of the 1990s have little to distinguish them from earlier periods. There is nothing new about disagreements over the best ways to educate the nation's school children. The periodic waves of education reform from the nation's colleges of education are more similar than they are

different. The American education establishment has consistently advocated a progressivist education agenda for the bulk of the 20th century, and the mainstream views of the education community have enjoyed a commanding influence on public schools.[2] Recognizing this dominion in the early part of the century, William Bagley in 1926 lamented:

> In no other country are the professional students of education so influential. In no other country is school practice so quickly responsive to the suggestions emanating from this group. We may stigmatize our schools as "static, " "reactionary," "slow to change,"--reluctant to adopt what we, in our wisdom, prescribe. But compared to other countries, ours is the educational expert's paradise.[3]

Colleges of education exert powerful direct influence on elementary and middle school teachers, and indirect influence on them through other organizations such as state level departments of education and professional teacher organizations. The influence on high school math teachers, while still powerful, has been less direct because of the subject matter specialization of the high school curriculum. The content demands of mathematics itself have limited the direct influence of some pedagogical fashions on high school math teachers. However, because of the hierarchical nature of mathematics and its heavy dependence at any level on prerequisites, high school and even college mathematics courses have at times been strongly affected by progressivist ideas, especially at the end of the 20th century.

The political struggles and policy changes in mathematics education in the 1980s, and especially the 1990s are the major topics of this chapter. However, the events of the final two decades of the 20th century are more easily understood in an historical context. Throughout the 20th century the "professional students of education" have militated for child centered discovery learning, and against systematic practice and teacher directed instruction. In some cases, progressivist math programs of the 1990s were intentionally without student textbooks, since books might interfere with student discovery. The essence of the dictum from educators of the 1990s and late 1980s, that the teacher should be "a guide on the side and not a sage on the stage," was already captured in a statement from the principal of one of John Dewey's "schools of tomorrow" from the 1920s:

> The teacher's arbitrary assignment of the next ten pages in history, or nine problems in arithmetic, or certain descriptions in geography, cannot be felt by the pupil as a real problem and a personal problem.[4]

The next section provides a brief overview of some of the important historical trends and policies leading up to the events of the 1980s and 90s.

Historical Outline: 1920 to 1980

It would be a mistake to think of the major conflicts in education as disagreements over the most effective ways to teach. Broadly speaking, the education wars of the past century are best understood as a protracted struggle between content and pedagogy. At first glance, such a dichotomy seems unthinkable. There should no more be conflict between content and pedagogy than between one's right foot and left foot. They should work in tandem toward the same end, and avoid tripping each other. Content is the answer to the question of what to teach, while pedagogy answers the question of how to teach.

The trouble comes with the first step. Do we lead with the right foot or the left? If content decisions come first, then the choices of pedagogy may be limited. A choice of concentrated content precludes too much student centered, discovery learning, because that particular pedagogy requires more time than stiff content requirements would allow. In the same way, the choice of a pedagogy can naturally limit the amount of content that can be presented to students. Therein lies the source of the conflict.

With roots going back to Jean Jacques Rousseau and with the guidance of John Dewey, progressive education has dominated American schools since the early years of the 20th century. That is not to say that progressive education has gone unchallenged.[5] Challenges increased in intensity starting in the 1950s, waxed and waned, and in the 1990s gained unprecedented strength. A consequence of the domination of progressivism during the first half of the 20th century was a predictable and remarkably steady decrease of academic content in public schools.

The prescriptions for the future of mathematics education were articulated early in the 20th century by one of the nation's most influential education leaders, William Heard Kilpatrick. According to E. D. Hirsch, Kilpatrick was "the most influential introducer of progressive ideas into American schools of education."[6] Kilpatrick was an education professor at Teachers College at Columbia University, and a protege of John Dewey. According to Dewey, "In the best sense of the words, progressive education and the work of Dr. Kilpatrick are virtually synonymous."[7] Kilpatrick majored in mathematics at Mercer College in Macon, Georgia. His mathematical education included some graduate work at Johns Hopkins University, but his interests changed and he eventually attended Teachers College and joined the faculty in 1911. In his 27 years at Teachers College, he taught some 35,000 students and was described by the New York Post as "the million dollar professor" because the fees paid by his students to the college exceeded this amount. In some instances there were more than 650 students in a single one of his auditorium sized classes.[8] His book, *Foundations of Method*, written in 1925 became a standard text for teacher education courses across the country.

Reflecting mainstream views of progressive education, Kilpatrick rejected the notion that the study of mathematics contributed to mental discipline. His view was that subjects should be taught to students based on their direct practical value, or if students independently wanted to learn those subjects. This point of view toward education comported well with the pedagogical methods endorsed by progressive education. Limiting education primarily to utilitarian skills sharply limited academic content, and this helped to justify the slow pace of student centered, discovery learning, the centerpiece of progressivism. Kilpatrick proposed that the study of algebra and geometry in high school be discontinued "except as an intellectual luxury." According to Kilpatrick, mathematics is "harmful rather than helpful to the kind of thinking necessary for ordinary living." In an address before the student body at the University of Florida, Kilpatrick lectured, "We have in the past taught algebra and geometry to too many, not too few."[9]

Progressivists drew support from the findings of psychologist Edward L. Thorndike. Thorndike conducted a series of experiments beginning in 1901 that cast doubt on the value of mental discipline and the possibility of transfer of training from one activity to another. These findings were used to challenge the justification for teaching mathematics as a form of mental discipline and contributed to the view that any mathematics education should be for purely utilitarian purposes.[10] Thorndike stressed the importance of creating many "bonds" through repeated practice and championed a stimulus-response method of learning. This led to the fragmentation of arithmetic and the avoidance of teaching closely related ideas too close in time, for fear of establishing incorrect bonds. According to one writer, "For good or for ill, it was Thorndike who dealt the final blow to the 'science of arithmetic.'"[11]

Kilpatrick's opinion that the teaching of algebra should be highly restricted was supported by other experts. According to David Snedden, the founder of educational sociology, and a prominent professor at Teachers College at the time, "Algebra...is a nonfunctional and nearly valueless subject for 90 percent of all boys and 99 percent of all girls--and no changes in method or content will change that."[12] During part of his career, Snedden was Commissioner of Education for the state of Massachusetts.[13]

In 1915 Kilpatrick was asked by the National Education Association's Commission on the Reorganization of Secondary Education to chair a committee to study the problem of teaching mathematics in the high

schools. The committee included no mathematicians and was composed entirely of educators.[14] Kilpatrick directly challenged the use of mathematics to promote mental discipline. He wrote, "No longer should the force of tradition shield any subject from scrutiny...In probably no study did this older doctrine of mental discipline find larger scope than in mathematics, in arithmetic to an appreciable extent, more in algebra, and most of all in geometry."[15] Kilpatrick maintained in his report, *The Problem of Mathematics in Secondary Education*, that nothing in mathematics should be taught unless its probable value could be shown, and recommended the traditional high school mathematics curriculum for only a select few.[16]

It was not surprising that mathematicians would object to Kilpatrick's report as an attack against the field of mathematics itself. David Eugene Smith, a mathematics professor at Teachers College and renowned historian of mathematics, tried to stop the publication of Kilpatrick's report as a part of the *Cardinal Principles of Secondary Education*, the full report of the Commission on the Reorganization of Secondary Education, and one of the most influential documents for education in the 20th century. Smith charged that there had been no meeting of the math committee and that Kilpatrick was the sole author of the report. Moreover, Kilpatrick's committee was not representative of teachers of mathematics or of mathematicians.[17]Nevertheless, Kilpatrick's report was eventually published in 1920 by the U.S. Commissioner of Education, Philander P. Claxton, a friend of Kilpatrick.[18]

The Kilpatrick committee and leading educational theoreticians had thrown the gauntlet, and the Mathematical Association of America (MAA) responded vigorously. Already in 1916, in anticipation of the Kilpatrick report, E. R. Hedrick, the first president of the MAA, appointed a committee called the National Committee on Mathematical Requirements. It was chaired by J. W. Young of Dartmouth and included mathematicians E. H. Moore, Oswald Veblen, and David E. Smith, in addition to several prominent teachers and administrators from the secondary school system. The reports of this committee were delayed because of World War I, but they were eventually collected into a 625 page volume entitled, *The Reorganization of Mathematics for Secondary Education*. The report was published in 1923 and is sometimes referred to as the *1923 Report*.

Meanwhile in 1920, the National Council of Teachers of Mathematics (NCTM) was founded, largely at the instigation of the MAA. The first NCTM president, C. M. Austin, made it clear that the organization would "keep the values and interests of mathematics before the educational world" and he urged that "curriculum studies and reforms and adjustments come from the teachers of mathematics rather than from the educational reformers." The NCTM was created in part to counter the progressivist educational agenda for mathematics, and it later played an important role in disseminating the *1923 Report*.[19]

The *1923 Report*was perhaps the most comprehensive ever written on the topic of school mathematics. It included an extensive survey of secondary school curricula, and it documented the training of mathematics teachers in other countries. It discussed issues related to the psychology of learning mathematics, and justified the study of mathematics in terms of its applications as well as its intrinsic value. It even proposed curricula for the schools. In contradiction to the Kilpatrick report, the *1923 Report* underscored the importance of algebra to "every educated person."[20] The *1923 Report* exerted some influence on public education. For example, some of the policies of the College Examination Board were based upon recommendations in the *1923 Report*. However, over the next two decades, the views expressed in the Kilpatrick report wielded greater influence than the *1923 Report*.[21] The NCTM also changed over time. It grew and gradually it "attracted to its membership and to its leadership those in positions much more subject to the influence and pressure of the professional reform movements."[22]

In the 1930s the education journals, textbooks, and courses for administrators and teachers advocated the major themes of progressivism. The school curriculum would be determined by the needs and interests of children, as determined by professional educators, and not by academic subjects. It became a cliche in the

1930s, just as in the 1990s, for educators to say, "We teach children, not subject matter." The Activity Movement of the 1930s promoted the integration of subjects in elementary school, and argued against separate instruction in mathematics and other subjects. It drew its inspirations from Kilpatrick's writings. The Activity Movement spread rapidly into the nation's elementary schools. High schools were more resistant in part because the teachers were trained in specific subject areas and they were less willing to discard their specialties in favor of an ill defined holism. Some proponents of the Activity Movement did not even acknowledge that reading and learning the multiplication tables were legitimate activities . As in the 1990s, there was public resistance to the education doctrines of this era. Among the critics were Walter Lippman, one of the nation's most widely respected commentators on public affairs, and literary critic, Howard Mumford Jones.[23]

In the 1940s it became something of a public scandal that army recruits knew so little math that the army itself had to provide training in the arithmetic needed for basic bookkeeping and gunnery.[24] Admiral Nimitz complained of mathematical deficiencies of would-be officer candidates and navy volunteers. The basic skills of these military personnel should have been learned in the public schools but were not.[25] As always, education doctrines did not sit well with much of the public. Nevertheless, by the mid-1940s, a new educational program called the Life Adjustment Movement emerged from the education community. The basic premise was that secondary schools were "too devoted to an academic curriculum." Education leaders presumed that 60% or more of all public school students lacked the intellectual capability for college work or even for skilled occupations, and those students would need a school program to prepare them for every day living. They would need appropriate high school courses, including math programs, that focused purely on practical problems such as consumer buying, insurance, taxation, and home budgeting, but not on algebra, geometry, or trigonometry. The students in these courses would become unskilled or semiskilled laborers, or their wives, and they would not need an academic education. Instead they would be instructed in "home, shop, store, citizenship, and health."

By 1949 the Life Adjustment Movement had substantial support among educators, and was touted by numerous federal and state education agencies. Some educators even suggested that in order to avoid stigmatizing the students in these programs, non-academic studies should be available to all students. Life Adjustment could meet the needs of all American students.[26]

However, many schools stubbornly clung to the teaching of academic subjects even when they offered life adjustment curricula as well. Moreover, parents of school children resisted these changes; they wanted their own children educated and not merely adjusted. They were sometimes joined by university professors and journalists who criticized the lack of academic content of the progressivist life adjustment programs. Changes in society at large also worked against the life adjustment agenda. Through the 1940s, the nation had witnessed tremendous scientific and engineering advances. By the end of the decade, the appearance of radar, cryptography, navigation, atomic energy, and other technological wonderments changed the economy and underscored the importance of mathematics in the modern world. This in turn caused a recognition of the importance of mathematics education in the schools. By the end of the 1940s, the public school system was the subject of a blizzard of criticisms, and the life adjustment movement fizzled out. Among the critics was Mortimer Smith. Reminiscent of Bagley's 1926 characterization of "students of education," he wrote in his 1949 book *Madly They Teach:*

> ...those who make up the staffs of the schools and colleges of education, and the administrators and teachers whom they train to run the system, have a truly amazing uniformity of opinion regarding the aims, the content, and the methods of education. They constitute a cohesive body of believers with a clearly formulated set of dogmas and doctrines, and they are perpetuating the faith by seeing to it through state laws and the

> rules of state departments of education, that only those teachers and administrators are certified who have been trained in the correct dogma.[27]

As would be the case in the final decade of the century, critics of this period complained of a lack of attention to basic skills.[28]

Progressive education was forced into retreat in the 1950s, and even became the butt of jokes and vitriol.[29] During the previous half century, enrollment in advanced high school mathematics courses, and other academic subjects, had steadily decreased, thanks at least in part to progressive education. From 1933 to 1954 not only did the percentage of students taking high school geometry decrease, even the actual numbers of students decreased in spite of soaring enrollments. The following table gives percentages of high school students enrolled in high school math courses.[30]

Percentages of U.S. High School Students Enrolled in Various Courses

School Year	Algebra	Geometry	Trigonometry
1909 to 1910	56.9%	30.9%	1.9%
1914 to 1915	48.8%	26.5%	1.5%
1921 to 1922	40.2%	22.7%	1.5%
1927 to 1928	35.2%	19.8%	1.3%
1933 to 1934	30.4%	17.1%	1.3%
1948 to 1949	26.8%	12.8%	2.0%
1952 to 1953	24.6%	11.6%	1.7%
1954 to 1955	24.8%	11.4%	2.6%

The "New Math" period came into being in the early 1950s and lasted through the decade of the 1960s. New Math was not a monolithic movement. According to a director of one of the first New Math conferences, "The inception of the New Math was the collision between skills instruction and understanding ...The disagreements between different entities of the New Math Movement were profound. Meetings between mathematicians and psychologists resulted only in determining that the two had nothing to say to each other."[31] However, in a 1960 paper delivered to the NCTM, Harvard psychologist Jerome Bruner wrote:

> I am struck by the fact that certain ideas in teaching mathematics that take a student away from the banal manipulation of natural numbers have the effect of freshening his eye to the possibility of discovery. I interpret such trends as the use of set theory in the early grades partly in this light--so too the Cuisenaire rods, the use of modular arithmetic, and other comparable devices.[32]

In spite of disagreements, most projects of that period shared some general features. The New Math groups introduced curricula that emphasized coherent logical explanations for the mathematical procedures taught in the schools. New Math was clearly a move away from the anti-intellectualism of the previous half-century of progressivist doctrine. For the first time, mathematicians were actively involved in contributing to K-12 school mathematics curricula.

The University of Illinois Committee on School Mathematics headed by Max Beberman began in 1951 and was the first major project associated with the New Math era. Beberman's group published a series of high school math textbooks, and drew financial support from the Carnegie Corporation and the U.S. Office of Education. In 1955, the College Entrance Examination Board established a Commission on Mathematics to investigate the "mathematics needs of today's American youth." The Commission,

consisting of high school teachers, math educators, and mathematicians, issued a report with recommendations for a curriculum to better prepare students for college, and produced a sample textbook for twelfth grade on probability and statistics.[33]The efforts of these and other early groups received little attention until the U.S.S.R launched *Sputnik*, the first space satellite, in the fall of 1957. The American press treated *Sputnik* as a major humiliation, and called attention to the low quality of math and science instruction in the public schools. Congress responded by passing the 1958 National Defense Education Act to increase the number of science, math, and foreign language majors, and to contribute to school construction.

That same year, the American Mathematical Society set up the School Mathematics Study Group (SMSG), headed by Edward G. Beg1e, then at Yale University, to develop a new curriculum for high schools. Among the many curriculum groups of the New Math period, SMSG was the most influential. It created junior and senior high school math programs and eventually elementary school curricula as well. The original eight members of SMSG were appointed by the president of the American Mathematical Society, but thereafter the two organizations had no formal connection. SMSG subsequently appointed a 26 member advisory committee and a 45 member writing group which included 21 college and university mathematicians as well as 21 high school math teachers and supervisors.[34]

The National Council of Teachers of Mathematics set up its own curriculum committee, the Secondary School Curriculum Committee, which came out with its recommendations in 1959. Many other groups emerged during this period including, the Ball State Project, the University of Maryland Mathematics Project, the Minnesota School Science and Mathematics Center, and the Greater Cleveland Mathematics Program. In the late 1950s, individual high school and college teachers started to write their own texts along the lines suggested by the major curriculum groups.[35]

One of the contributions of the New Math movement was the introduction of calculus courses at the high school level.[36] Although, there were important successes in the New Math period, some of the New Math curricula were excessively formal, with little attention to basic skills or to applications of mathematics. Programs that included treatments of number bases other than base ten, as well as relatively heavy emphases on set theory, or more exotic topics, tended to confuse and alienate even the most sympathetic parents of school children. There were instances in which abstractness for its own sake was overemphasized to the point of absurdity.[37] Many teachers were not well equipped to deal with the demanding content of the New Math curricula. As a result public criticisms increased.

A substantial number of mathematicians had already expressed serious reservations relatively early in the New Math period. In 1962, a letter entitled *On The Mathematics Curriculum Of The High School*, signed by 64 prominent mathematicians, was published in the *American Mathematical Monthly* and *The Mathematics Teacher*. The letter criticized New Math and offered some general guidelines and principles for future curricula.[38]

By the early 1970s New Math was dead. The National Science Foundation discontinued funding programs of this type, and there was a call to go "back to the basics" in mathematics as well as in other subjects.[39] However, this direction for education did not go unchallenged. Progressive education had recovered from its doldrums of the 1950s, and by the late 1960s and early 1970s, it had regained its momentum. A. S. Niell's book *Summerhill*, published in 1960, is an account of an ultra progressive school in England. It was one of the most influential books on education of that decade. Founded in 1921 in Suffolk, England as a boarding school for relatively affluent children, Summerhill students determined completely what they would learn, and when. Niell wrote, "Whether a school has or has not a special method for teaching long division is of no significance, for long division is of no importance except to those who *want* to learn it. And the child who *wants* to learn long division *will* learn it no matter how it is

taught." By 1970, some 200,000 copies of *Summerhill* were being sold per year, and it was required reading in 600 university courses.[40]

Modeled on Summerhill, and supported by the challenges at that time of structures of authority, both within education and the larger society, "free schools" proliferated, and eventually helped give rise to the Open Education Movement. The Open Education Movement was nothing new; it was just a repetition of progressivist programs promoted in the 1920s, but the idea of letting children decide each day what they should learn at activity tables, play corners, or reading centers, was once again promoted as profound and revolutionary.[41]

The effects of the Open Education Movement were particularly devastating to children with limited resources, due to their lack of access to supplemental education from the home, or tutoring in basic skills outside of school. Lisa Delpit, an African American educator who taught in an inner city school in Philadelphia in the early 1970s wrote about the negative effects of this type of education on African American children. Relating a conversation with another African American teacher, she explained, "White kids learn how to write a decent sentence. Even if they don't teach them in school, their parents make sure they get what they need. But what about our kids? They don't get it at home..." Summarizing the effects of the open classroom movement from her perspective in 1986, Professor Delpit wrote:

> I have come to believe that the "open classroom movement," despite its progressive intentions, faded in large part because it was not able to come to terms with the concerns of poor and minority communities.[42]

Another prominent educator, Nancy Ichinaga, came to similar conclusions about the effects of the Open Education Movement on low income students, based on her experience as principal of Bennett-Kew Elementary school, in Inglewood, California. Ichinaga began a 24 year career as principal of Bennett-Kew in the Fall of 1974, one month before scores from the California's standardized test were released. At that time the school included only grades K-3 and it was called Bennett Elementary school. Bennett's 1974 third grade students ranked at the third percentile in the state, almost the absolute bottom. The school was then in its fourth year of the "Open Structure Program" and the student body throughout her tenure as principal was nearly 100 percent minority and low income. Reacting with shock and dismay at the test scores, Ichinaga confronted the teachers who admitted that their program was not working. The entire student body was illiterate and the student centered mathematics program was in shambles.

With the collaboration of her teachers, Nancy Ichinaga introduced clearly defined and well structured reading and math programs which included practice in basic skills. After a few years, test scores increased to well beyond the 50th percentile, and by the end of the 20th century, her school had earned national acclaim and became a model for others to emulate.[43] At an education conference held in May 1999, Principal Ichinaga described the situation in her school in 1974:

> My school had been patterned after *Summerhill*. And that's how bad it was! The kids used to make jello and bake cookies, and I used to tell the teachers, "Do you know what you've accomplished? You just gave them rotten teeth!"[44]

As in earlier periods of the 20th century, the agenda of progressivist educators was resisted by broad sectors of the public. The majority of states created minimum competency tests in basic skills starting in the mid-1970s, and almost half of them required students to pass these tests as a condition for graduation from high school. Due to public demand, some school districts created "fundamental schools" that emphasized traditional academics and promoted student discipline. While basic skills tests held the Open Education Movement in check, by their nature they could not be used to hold students to very high standards, or to raise existing standards. During the 1970s, standardized test scores steadily decreased and bottomed out in the early 1980s.[45]

The 1980s: Prelude to National Standards

In the early 1980s, there was widespread recognition that the quality of math and science education had been deteriorating. A 1980 report by a presidential commission pointed to low enrollments in advanced mathematics and science courses and the general lowering of school expectations and college entrance requirements.[46] Among the various reports and commissions to investigate K-12 education in the early 1980s, two especially stand out: *An Agenda for Action* and *A Nation at Risk*. The different points of view and prescriptions for change expressed in these two reports characterize to some extent the opposing factions in the math wars of the 1990s.

The National Council of Teachers of Mathematics released *An Agenda for Action* in 1980. The report called for new directions in mathematics education which would later be codified in 1989 in the form of national standards. *An Agenda for Action* recommended that problem solving be the focus of school mathematics in the 1980s, along with new ways of teaching. The report asserted that "Requiring complete mastery of skills before allowing participation in challenging problem solving is counterproductive, " and "Difficulty with paper-and-pencil computation should not interfere with the learning of problem-solving strategies." Technology would make problem solving available to students without basic skills. According to the report, "All students should have access to calculators and increasingly to computers throughout their school mathematics program." This included calculators "for use in elementary and secondary school classrooms." The report also warned, "It is dangerous to assume that skills from one era will suffice for another," and called for "decreased emphasis on such activities as...performing paper and pencil calculations with numbers of more than two digits." This would be possible because "The use of calculators has radically reduced the demand for some paper-and-pencil techniques." The report also recommended that "Team efforts in problem solving should be common place in elementary school classrooms," and encouraged "the use of manipulatives, where suited, to illustrate or develop a concept or skill." *An Agenda for Action* also called for "a wider range of measures than conventional testing." All of these directions would later become issues of contention in the math wars of the 1990s.[47]

Perhaps the boldest and most far reaching recommendation of *An Agenda for Action* was its proposal for "Mathematics educators and college mathematicians" to "reevaluate the role of calculus in the differentiated mathematics programs." The report argued that "Emerging programs that prepare users of mathematics in nontraditional areas of application may no longer demand the centrality of calculus that has traditionally been demanded for all students." The de-emphasis of calculus, when carried out on a large enough scale, would support the move away from the systematic development of the prerequisites of calculus: algebra, geometry, and trigonometry. The so-called "integrated" high school math books of the 1990s contributed to this tendency. While those books contained parts of algebra, geometry, and trigonometry, the developments of these traditional subjects were not systematic, and often depended on student "discoveries" that were incidental to solving "real world problems."

In spite of the NCTM's enthusiasm for the objectives of *An Agenda for Action*, the report received little attention. It was largely eclipsed by the 1983 report, *A Nation At Risk*.[48] This report was written by a commission appointed by Terrell Bell, the U.S. Secretary of Education, at that time. Unlike previous education reform efforts and reports by prestigious governmental bodies, this one captured the attention of the public. *A Nation At Risk* warned, "Our nation is at risk...the educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people." Even sharper was the statement, "If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exits today, we might well have viewed it as an act of war."

A Nation at Risk addressed a wide variety of education issues, including specific shortcomings in mathematics education. Regarding remedial mathematics instruction, the report found that:

> Between 1975 and 1980, remedial mathematics courses in public 4-year colleges increased by 72 percent and now [in 1983] constitute one-quarter of all mathematics courses taught in those institutions.
>
> Business and military leaders complain that they are required to spend millions of dollars on costly remedial education and training programs in such basic skills as reading, writing, spelling, and computation.

Although the authors of *A Nation at Risk* did not attempt to analyze the causes of these deficiencies, the lack of attention to basic skills in elementary schools caught up in the Open Education Movement of the late 1960s and early 1970s surely contributed to the need for more remedial courses in the 1980s in high school and college.

A Nation at Risk described high school course offerings as a "curricular smorgasbord" and reported, "We offer intermediate algebra, but only 31 percent of our recent high school graduates complete it; we offer French I, but only 13 percent complete it; and we offer geography, but only 16 percent complete it. Calculus is available in schools enrolling about 60 percent of all students, but only 6 percent of all students complete it."

The importance of student assessment was also addressed. The report envisioned a role for standardized tests that foreshadowed a movement toward accountability in the late 1990s[49]:

> Standardized tests of achievement (not to be confused with aptitude tests) should be administered at major transition points from one level of schooling to another and particularly from high school to college or work. The purposes of these tests would be to: (a) certify the student's credentials; (b) identify the need for remedial intervention; and (c) identify the opportunity for advanced or accelerated work. The tests should be administered as part of a nationwide (but not Federal) system of State and local standardized tests. This system should include other diagnostic procedures that assist teachers and students to evaluate student progress.

A Nation at Risk called attention to the quality of teachers and complained, "Too many teachers are being drawn from the bottom quarter of graduating high school and college students." Teacher training programs were also criticized in the report:

> The teacher preparation curriculum is weighted heavily with courses in "educational methods" at the expense of courses in subjects to be taught. A survey of 1,350 institutions training teachers indicated that 41 percent of the time of elementary school teacher candidates is spent in education courses, which reduces the amount of time available for subject matter courses.

The report also drew attention to teacher shortages, especially math and science teachers:

> The shortage of teachers in mathematics and science is particularly severe. A 1981 survey of 45 States revealed shortages of mathematics teachers in 43 States, critical shortages of earth sciences teachers in 33 States, and of physics teachers everywhere.

A Nation at Risk also addressed the question of textbooks, proposing that they be upgraded to include more rigorous content. It called upon "university scientists, scholars, and members of professional societies, in collaboration with master teachers, to help in this task, as they did in the post-Sputnik era.

They should assist willing publishers in developing the products or publish their own alternatives where there are persistent inadequacies." The report addressed the textbook adoption process as well, suggesting that:

> In considering textbooks for adoption, States and school districts should: (a) evaluate texts and other materials on their ability to present rigorous and challenging material clearly; and (b) require publishers to furnish evaluation data on the material's effectiveness.

With widespread public concern about education, the release of *A Nation at Risk* resulted in newspaper headlines across the country. A number of states created task forces and commissions to measure their own state programs against the recommendations of *A Nation at Risk*.[50] It is illuminating to compare these recommendations to the California mathematics education polices of the late 1990s. As described below, California's mathematics policies in 1998 became the leading obstacle to progressivist domination in mathematics education. Yvonne Larson, the vice-chair of the Commission that released *A Nation at Risk*in 1983, served as the president of the California State Board of Education 1997. Whether by intent or coincidence, the California policies conformed rather well with a number of the recommendations of the 1983 report.

The 1989 NCTM Standards

With public opinion in support of a strong focus on basic skills and clear high standards, the NCTM took steps to recast its own agenda under the label of standards. In 1986 the NCTM established the Commission on Standards for School Mathematics. The *Curriculum and Evaluation Standards for School Mathematics* was developed during the summer of 1987 and revised in 1988 by four working groups whose members were appointed by John Dossey, the president of the NCTM at that time. During the 1987-88 school year, input was sought from classroom teachers across the country. The project was coordinated by Thomas A. Romberg. The final document was published in 1989, and during the following decade it was commonly referred to as the *NCTM Standards*, or as the *Standards*.[51] Of the 24 working group members who had direct input into the writing of the *Standards,*none were mathematicians, and only two were concurrent K-12 teachers; the remainder were, for the most part, teacher education professors and instructors from universities. However, the NCTM successfully promoted the *Standards*as if they were developed through a grass-roots, bottom-up process.[52]

The *NCTM Standards* were not standards in the usual sense of the word. Harold Stevenson, a psychologist at the University of Michigan, described them as follows:

> In our view the *NCTM standards*present a vague, somewhat grandiose, readily misinterpreted view of what American children should learn in mathematics. Moreover, the view fails to meet what we would consider to be the meaning of "standards." Standards should involve a progression of accomplishments or competencies that are to be demonstrated at defined times in the child's schooling. The *NCTM standards* give no indication (beyond four-year intervals) of the sequence with which the content is to be presented and are not helpful to the classroom teacher in designing lessons that meet the standards.
>
> The *NCTM standards* list goals with which no one would be likely to disagree. Of course we want children to value mathematics, to be mathematics problem solvers, to be confident of their ability, and to be able to reason and communicate mathematically. Certainly students must develop a number sense, have concepts of whole number operations, and the other kinds of skills and knowledge indicated under NCTM's

> curriculum standards. But the published standards do not integrate these two important components: the general attitudes and mathematical skills.[53]

The1989 NCTM *Curriculum and Evaluation Standards for School Mathematics* is comprised of sections devoted to general standards for the bands of grades: K-4, 5-8, and 9-12. Another section is devoted to "Evaluation Standards." In many respects, the *1989 NCTM* standards promoted the views of *An Agenda for Action*, but with greater elaboration. The grade level bands included lists of topics that were to receive "increased attention" and lists of topics that should receive "decreased attention." For example, in the K-4 band, the *Standards* called for greater attention to "Meanings of operations," "Operation sense," "Mental computation," "Use of calculators for complex computation," "Collection and organization of data," "Pattern recognition and description," "Use of manipulative materials," and "Cooperative work."

Included on the list for decreased attention in the grades K-4 were "Complex paper-and-pencil computations," "Long division," "Paper and pencil fraction computation," "Use of rounding to estimate," "Rote practice," "Rote memorization of rules," and "Teaching by telling." For grades 5-8 the *Standards* were even more radical. The following were included on the list to be de-emphasized: "Relying on outside authority (teacher or an answer key)," Manipulating symbols," "Memorizing rules and algorithms," "Practicing tedious paper-and-pencil computations," "Finding exact forms of answers."

As in *An Agenda for Action*, the *1989 NCTM Standards* put strong emphasis on the use of calculators throughout all grade levels. On page 8, the *Standards* proclaimed, "The new technology not only has made calculations and graphing easier, it has changed the very nature of mathematics..." The NCTM therefore recommended that, "appropriate calculators should be available to all students at all times." The Standards did concede that "the availability of calculators does not eliminate the need for students to learn algorithms," and it did acknowledge the need for "some proficiency with paper and pencil algorithms." However, these concessions were not supported in the classroom scenarios, or other parts of the document.

The *NCTM Standards* reinforced the general themes of progressive education, dating back to the 1920s, by advocating student centered, discovery learning. The utilitarian justification of mathematics was so strong that both basic skills and general mathematical principles were to be learned almost invariably through "real world" problems. Mathematics for its own sake was not encouraged. The variant of progressivism favored by the NCTM during this time was called "constructivism" and the *NCTM Standards* were promoted under this banner.[54]

The term "constructivism" was adapted from cognitive psychology by educators, and its meaning in educational contexts is different from its use in psychology. E.D. Hirsch Jr. provided a useful definition in his book, *The Schools We Need: Why We Don't Have Them*, which begins as follows:

> "**Constructivism**" A psychological term used by educational specialists to sanction the practice of "self-paced learning" and "discovery learning." The term implies that only constructed knowledge--knowledge which one finds out for one's self--is truly integrated and understood. It is certainly true that such knowledge is very likely to be remembered and understood, but it is not the case, as constructivists imply, that *only* such self-discovered knowledge will be reliably understood and remembered. This incorrect claim plays on an ambiguity between the technical and nontechnical uses of the term "construct" in the psychological literature...

Hirsch elaborated further on the psychological meaning of constructivism in his book. A more general and technical discussion was given in a paper by John R. Anderson, Lynne M. Reder, Herbert A. Simon entitled, *Applications and Misapplications of Cognitive Psychology to Mathematics*

Education.[55] Criticisms of educational constructivism, as in this article, were not well received by the education community. In an address before the California State Board of Education in April 1997, Hirsch described the treatment of this paper. "After a so-called peer review,*Educational Researcher* turned down the article, and agreed to print only a section of its critique of situated learning. This decision would have been unremarkable except that the three authors of the article happened to be among the most distinguished cognitive scientists in the world, John Anderson and two other colleagues at Carnegie Mellon, Lynn Reder, and Herb Simon. The latter happens also to be a Nobel prize winner."[56]

Mathematics education leaders drew support for educational constructivism from the writings of Jean Piaget and Lev Semenovich Vygotsky. Piaget's ideas about developmental stages of learning, and Vygotsky's concept, "Zone of Proximal Development," seemed to be consistent with the child-centered, cooperative learning approaches to education long favored by colleges of education.

In the fall of 1989, President George H. W. Bush, then in his first year of office, was invited by the nation's governors to an education summit in Charlottesville, Virginia. A bipartisan call went out for national standards. Participants at the 1989 Education Summit made a commitment to make U.S. students first in the world in mathematics and science by the year 2000.

Political leaders in the late 1980s were motivated by employers' complaints about the costs of teaching basic skills to entry level workers, and by the low standing of U.S. students in comparisons with foreign students in an era of economic competition.[57] The timing for the *NCTM Standards* could not have been better. The nation was looking for benchmarks that could improve education. The *NCTM Standards* had just been published, and by default they became the national model for standards. The *NCTM Standards* were immediately and perfunctorily endorsed by a long list of prominent organizations such as the American Mathematical Society, the Mathematical Association of America, and the Council of Scientific Society Presidents.

Within a few years, the NCTM produced two additional documents as part of its standards. One published in 1991 was narrowly focused on pedagogy and the other, published in 1995, was focused on testing.[58] By 1997 most state governments had adopted mathematics standards in close alignment with the NCTM standards.[59]

The National Science Foundation

The National Science Foundation (NSF) was the key to the implementation of the *NCTM Standards* across the nation. Without the massive support it received from the NSF, the sole effect of the *NCTM Standards* would have been to collect dust on bookshelves. Spurred by the 1989 Education Summit attended by President Bush and all of the nation's governors, the Education and Human Resources Division (EHR) of the NSF set about to make systemic changes in the way math and science were taught in U.S. schools. The blueprint for change in mathematics would be the *NCTM Standards*.

The NSF proceeded purposefully. The EHR developed a series of Systemic Initiative grants to promote fundamental changes in science and mathematics education in the nation's schools. The Statewide Systemic Initiatives were launched in 1991. These grants were designed in part to encourage state education agencies to align their state mathematics standards to the *NCTM Standards*. The result was a remarkable uniformity and adherence to the *NCTM Standards* at the state level.[60]

Recognizing that education is largely a matter of local control, the NSF also launched its Urban Systemic Initiative (USI) program in 1994. These USI grants were designed to implement the NCTM agenda at the school district level in large cities. The USI grants were followed by a program for Rural Systemic

Initiatives. By 1999, the USI had evolved into the Urban Systemic Program. This program allowed renewals of awards made under the USI program.

At first, the Systemic Initiative grants were awarded to proposals generally aligned to the educational views of the NSF, but awardees were allowed substantial freedom to develop their own strategies for reform. As the program evolved, so did the guidelines. By 1996, the NSF clarified its assumptions about what constitutes effective, standards-based education and asserted that[61]:

- *All children can learn by using and manipulating scientific and mathematical ideas that are meaningful and relate to real-world situations and to real problems.*
- *Mathematics and science are learned by doing rather than by passive methods of learning such as watching a teacher work at the chalkboard. Inquiry-based learning and hands-on learning more effectively engage students than lectures.*
- *The use and manipulation of scientific and mathematical ideas benefits from a variety of contributing perspectives and is, therefore, enhanced by cooperative problem solving.*
- *Technology can make learning easier, more comprehensive, and more lasting.*
- *This view of learning is reflected in the professional standards of the National Council of Teachers of Mathematics, the American Association for the Advancement of Science, and the National Research Council of the National Academy of Sciences.*

The NSF was clear in its support of the *NCTM Standards* and of progressive education. Children should learn through group-based discovery with the help of manipulatives and calculators. Earlier research funded by the NSF, such as "Project Follow Through," which reached very different conclusions about what works best in the classroom, would not be considered.[62]Regardless of what cognitive psychology might say about teaching methodologies, only constructivist programs would be supported.

Along with the Systemic Initiative awards, the NSF supported the creation and development of commercial mathematics curricula aligned to the *NCTM Standards*. In the decade of the 1990s, the National Science Foundation sponsored the creation of the following mathematics programs for K-12:

Elementary school

Everyday Mathematics (K-6)

TERC's Investigations in Number, Data, and Space (K-5)

Math Trailblazers (TIMS) (K-5)

Middle school

Connected Mathematics (6-8)

Mathematics in Context (5-8)

MathScape: Seeing and Thinking Mathematically (6-8)

MATHThematics (STEM) (6-8)

Pathways to Algebra and Geometry (MMAP) (6-7, or 7-8)

High school

Contemporary Mathematics in Context (Core-Plus Mathematics Project) (9-12)

Interactive Mathematics Program (9-12)

MATH Connections: A Secondary Mathematics Core Curriculum (9-11)

Mathematics: Modeling Our World (ARISE) (9-12)

SIMMS Integrated Mathematics: A Modeling Approach Using Technology (9-12)

The development of NCTM aligned mathematics programs for K-12 was of obvious importance to the NSF (for a list of math programs explicitly endorsed by the NCTM, see the Appendix). How could the NCTM agenda be carried out without classroom materials that were specifically aligned to the *NCTMStandards*? An important component of the Systemic Initiatives was the aggressive distribution of NCTM aligned curricula for classroom use. The *NCTM Standards* were vague as to mathematical content, but specific in its support of constructivist pedagogy, the criterion that mattered most to the NSF. It should be noted that the Systemic Initiatives sometimes promoted curricula not on the list above, such as College Preparatory Mathematics, a high school program, and MathLand, a K-6 curriculum. MathLand was one of the most controversial of the widely used programs aligned to the *NCTM Standards*.[63]

In addition to aligning state math standards to the *NCTM standards* and creating and distributing math books and programs aligned to those standards, the NSF attempted with considerable success to push these approaches up to the university level. Most notable in this regard was the NSF's funding of a "reform calculus" book, often referred to as "Harvard Calculus," that relied heavily on calculators and discovery work by the students, and minimized the level of high school algebra required for the program.[64]

The NSF also funded distribution centers to promote the curricular programs it had helped to create. For example, an NSF sponsored organization created in 1997 called, "The K-12 Mathematics Curriculum Center," had a mission statement "to support school districts as they build an effective mathematics education program using curriculum materials developed in response to the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics*."

The Education and Human Resources Division of the NSF faced a serious hurdle in carrying out its Systemic Initiatives. U.S. K-12 education collectively was a multi-billion dollar operation and the huge budgets alone gave public education an inertia that would be hard to overcome. Even though the millions of dollars at its disposal made the EHR budget large in absolute terms, it was miniscule relative to the combined budgets of the school systems that the NSF sought to reform. It would not be easy to effect major changes in K-12 mathematics and science education without access to greater resources.

To some extent private foundations contributed to the goal of implementing the *NCTM Standards* through teacher training programs for the curricula supported the by the NSF, and in other ways. The Noyce Foundation was especially active in promoting NCTM aligned math curricula in Massachusetts and parts of California. Others such as the W. M. Keck Foundation and Bank of America contributed as well. However, the NSF itself found ingenious ways to increase its influence. The strategy was to use small grants to leverage major changes in states and school districts. NSF Assistant Director Luther Williams,

who was in charge of the Education and Human Resources Division, explained the strategy in a July 1998 Urban Systemic Initiative Summary Update:

> The NSF investment that promotes systemic reform will never exceed a small percentage of a given site's overall budget. The "converged" resources are not merely fiscal, but also strategic, in that they help induce a unitaryÖ reform operation. The catalytic nature of the USI-led reform obligates systemwide policy and fiscal resources to embrace standards-based instruction and create conditions for helping assorted expenditures to become organized and used in a single-purpose direction.

NSF Assistant Director Williams gave successful examples of this strategy. "Cleveland devoted half of its available bond referendum funding" for USI-related instructional material. "Los Angeles is one of several cities in the USI portfolio that places all Title II funding resources under the control of the USI." "In the Fresno Unified School System, $31 million of Title 1 funds have been realigned in support of USI activities."[65]

The Systemic Initiatives were extraordinarily successful in promoting the *NCTM Standards* and implementing NCTM aligned curricula at the classroom level. Los Angeles Unified School District (LAUSD), the second largest school district in the nation, serves as an illustrative example.

LAUSD was awarded a five year Urban Systemic Initiative grant in 1995 for $15 million. The $3 million per year from the Los Angeles Systemic Initiative (LASI) amounted to only one-twentieth of one percent of LAUSD's annual budget of $5.8 billion, or about $3.79 per student per year in the district. Yet, the LASI project exerted almost complete control over mathematics and science education in the district. In addition to Title II funds, LASI gained control of the school district's television station and its ten science and technology centers. According to Luther Williams' July 1998 Summary update, "[LASI] accountability became the framework for a major policy initiative establishing benchmarks and standards in all subject areas for the entire school system." LASI developed the district standards not only for math and science, but also English and social studies. All four sets of standards were adopted by the school district in 1996.

The Los Angeles School district math standards were so weak and vague that they were a source of controversy. One typical standard, without any sort of elaboration, asked students to "make connections among related mathematical concepts and apply these concepts to other content areas and the world of work." The LASI/LAUSD standards stipulated the use of calculators and "other appropriate technology" before the end of third grade, thus raising the possibility that students would not be required to master arithmetic. The word "triangle" did not even appear in the standards at any grade level. By design, trigonometry and all Algebra II topics were completely missing.[66] Like the *NCTM Standards*, the LAUSD/LASI standards were given only for bands of grades, rendering them at best useless, even if they had been otherwise competently written.

The 1996 LAUSD/LASI math standards paved the way for the dissemination of textbooks and curricula aligned to those standards, as well as staff development in their use. The LASI 1997 annual report explained:

> LAUSD's urban systemic initiative is well under way with its efforts to renew and unify districtwide instruction using standards-based curricula. These curricula are characterized by hands-on, inquiry based, problem solving, integrated/coordinated, student-teacher interactive instruction in math, science, technology for grades K-12. These efforts are supported and strengthened by needs-based staff development, increased communication among teachers and staff, changes in administrative policies that are essential for student

access to the systemic benefits, and checks on progress and process at preselected gates in the systemís superstructure.

LASI was successful in distributing "hands-on, inquiry based" curricula aligned to the *NCTM Standards* to LAUSD schools. LASI specifically recommended NCTM aligned curricula for all grades, including MathLand for K-5. By July 1998, more than half of all LAUSD schools were using math curricula aligned to the *NCTM Standards*, and LASI publicly announced its plan to require all LAUSD high schools to use one of four "integrated math" curricula within five years: Core-Plus, Interactive Mathematics Program, College Preparatory Mathematics, or McDougal Littell's "Integrated Math."[67] Two of these were funded by the NSF. This plan was not carried out because of the adoption of a new set of mathematics standards by the state of California in December 1997. But long after these rigorous California State Mathematics Standards were adopted, LAUSD schools continued to use LASI endorsed material. At a meeting of the LAUSD school board on May 2, 2000, it was revealed that fewer than three percent of elementary schools in the district were using California state approved mathematics programs. MathLand was used by 45 percent of the 420 elementary schools in LAUSD, while Quest 2000, a similar NCTM aligned program, was used by another 24 percent of the district's elementary schools. Eventually, the 1997 California mathematics standards were accepted and implemented by LAUSD, but not before a generation of students was educationally disenfranchised by the NSF Systemic Initiative Program.

The NSF's Systemic Initiative programs in other parts of the country were similarly successful in promoting mathematics curricula funded by the NSF, or otherwise aligned to the *NCTM Standards*. El Paso, Texas serves as an example. The El Paso Urban Systemic Initiative grant was awarded in 1994 and administered under the direction of the El Paso Collaborative for Academic Excellence. This collaborative coordinated other NSF funded projects including the Partnership for Excellence in Teacher Education and Model Institutions for Excellence, as well as private foundation grants, including support from The Exxon Corporation, The Pew Charitable Trusts, and The Coca-Cola Foundation.

El Paso is geographically removed from other U.S. cities and is unusual in that it is a "closed system." The teachers trained at the University of Texas, El Paso (UTEP) teach almost exclusively in the El Paso school districts, and the teachers in the El Paso school districts almost exclusively undertook their university studies at UTEP. This made the effectiveness of the K-12 and university programs easier to assess. It also made the entire education system easier to control. During the 1990s, the K-16 education system in El Paso was highly coordinated and focused on implementing constructivist math and science education programs. For this reason, it became a model center for educators from other parts of the country to visit and study.[68]

The Collaborative in El Paso worked in close coordination with the Texas Statewide Systemic Initiative housed in the Charles A. Dana Center in Austin. The Texas SSI developed an*Instructional Materials Analysis and SelectionScoring Grid* for Texas school districts to use in selecting math textbooks. The recommended criteria for selecting K-8 mathematics curricula included:

- Materials provide opportunities for teaching students to work in collaborative and cooperative groups
- Materials provide opportunity for the appropriate use of technology
- Students are engaged in the development of mathematical understanding through the use of manipulatives
- Multiple forms of assessment activities, such as student demonstrations, rubrics, self-reflections, observations, and oral and written work are used throughout the instructional materials
- Technology is built into the assessment tools
- Assessment activities take into account the ways in which students' unique qualities influence how they learn and how they communicate their understanding

- The instructional materials reflect cultural diversities and address historical perspectives
- Problem solving permeates the entire instructional material through investigative situations

The curricula chosen for El Paso public schools by 1999 were all NSF sponsored: TERC's Investigations in Number, Data, and Space (K-5), Connected Mathematics (6-8), SIMMS Integrated Mathematics: A Modeling Approach Using Technology (9-12).

The El Paso Collaborative for Academic Excellence created a confidential student evaluation questionnaire to monitor teaching methods used in high school math classrooms in all of EL Paso's public high schools. The evaluation included the following questions to students:

- How often do MOST STUDENTS talk with each other to describe or justify the strategy they used to solve a problem?
- How often does THE CLASS go in depth on a few problems instead of covering a large number of problems in the class period?
- How often does THE TEACHER TALK during most of the period?
- How often do YOU show that you understand a solution to a problem by explaining it in writing?
- How often do YOU use math in science and science in math?
- How often does THE CLASS work in pairs or groups to explain solutions?
- How often do YOU use hand calculators or computers to analyze data or solve problems?

The NSF awarded the Texas Statewide Systemic Initiative $2 million per year beginning in 1994. Yet, in spite of the low funding, the Texas SSI "provides leadership for a vast array of agency partnerships, and influences all aspects of education in Texas. Curricula, instructional practices, textbooks, assessment, professional development of teachers, teacher evaluation, teacher certification, and preservice teacher education all now fall under the purview of the Texas SSI."[69]

Public Resistance to the NCTM Standards

To understand the public backlash against the NCTM math programs of the1990s, one needs to understand some of the mathematical shortcomings of these programs. The mathematics books and curricula that parents of school children resisted shared some general features. Those programs typically failed to develop fundamental arithmetic and algebra skills. Elementary school programs encouraged students to invent their own arithmetic algorithms, while discouraging the use of the superior standard algorithms for addition, subtraction, multiplication, and division. Calculator use was encouraged to excess, and in some cases calculators were even incorporated into kindergarten lesson plans. Student discovery group work was the preferred mode of learning, sometimes exclusively, and the guidelines for discovery projects were at best inefficient and often aimless. Topics from statistics and data analysis were redundant from one grade level to the next, and were overemphasized. Arithmetic and algebra were radically de-emphasized. Mathematical definitions and proofs for the higher grades were generally deficient, missing entirely, or even incorrect. Some of the elementary school programs did not even provide books for students, as they might interfere with student discovery. Written and published criticisms from many sources, including mathematicians, of specific mathematics programs were widespread in the 1990s and reinforced the convictions of dissatisfied parents.[70]

But not everyone viewed the near absence of the standard algorithms of arithmetic in NCTM aligned books as a shortcoming. Some prominent educational researchers were explicit in their opposition to the teaching of algorithms to children. An article in the 1998 Yearbook of the NCTM entitled, *The Harmful Effects of Algorithms in Grades 1-4* by Constance Kamii and Ann Dominick provides examples. Citing earlier education research, the authors wrote, "By the 1980s, some researchers were seriously questioning

the wisdom of teaching conventional algorithms," and then listed examples of such research. Tracing the history of this line of inquiry they added, "Some investigators went further in the 1990s and concluded that algorithms are harmful to children," with examples provided. Elaborating, they wrote:

> Piaget's constructivism, and the more than sixty years of scientific research by him and others all over the world led Kamii to a compelling hypothesis: Children in the primary grades should be able to invent their own arithmetic without the instruction they are now receiving from textbooks and workbooks. This hypothesis was amply verified...

Kamii co-authored another article in the 1999 Yearbook of the NCTM in which similar conclusions were reached about the algorithms for the arithmetic of fractions.[71]

Opposition to conventional arithmetic algorithms was not restricted to academic researchers. Similar convictions were held by teacher trainers with substantial influence. In a 1994 article entitled, *Arithmetic: The Last Holdout*, Marilyn Burns wrote:

> I am a teacher who has embraced the call for change completely. I've made shifts in my teaching so that helping children learn to think, reason, and solve problems has become the primary objective of my math instruction...I do not give timed tests on basic facts. I make calculators available for students to use at all times. I incorporate a variety of manipulative materials into my instruction. I do not rely on textbooks because textbooks, for the most part, encourage "doing the page" rather than "doing mathematics."[72]

Parents of school children in the 1990s were directly confronted by policies based on these ideas. For example, the Los Angeles Times reported in 1997:

> One missionary in the Reform cause is consultant Ruth Parker, who rejects long division and multiplication tables as nonsensical leftovers from a pre-calculator age. She urges audiences to "let kids play with numbers," and they will figure out most any math concept. Parker has spoken before 20,000 people over the last six months at the behest of school districts.[73]

Parents who worried that their children were getting unsound educations from NCTM aligned mathematics programs did not give much credence to education research findings or the advice of education experts, and most mathematicians didn't either. Perhaps the general attitude of parents was best captured by Jaime Escalante, the nationally famous mathematics teacher immortalized in the film *Stand and Deliver*, when he said, "whoever wrote [the *NCTM Standards*] must be a physical education teacher." [74]

Sifting through the claims and counterclaims, journalists of the 1990s tended to portray the math wars as an extended disagreement between those who wanted basic skills versus those who favored conceptual understanding of mathematics. The parents and mathematicians who criticized the NCTM aligned curricula were portrayed as proponents of basic skills, while educational administrators, professors of education, and other defenders of these programs, were portrayed as proponents of conceptual understanding, and sometimes even "higher order thinking." This dichotomy is implausible. The parents leading the opposition to the NCTM Standards, as discussed below, had considerable expertise in mathematics, generally exceeding that of the education professionals. This was even more the case of the large number of mathematicians who criticized these programs. Among them were some of the world's most distinguished mathematicians, in some cases with mathematical capabilities near the very limits of human ability. By contrast, many of the education professionals who spoke of "conceptual understanding" lacked even a rudimentary knowledge of mathematics.

More fundamentally, the separation of conceptual understanding from basic skills in mathematics is misguided. It is not possible to teach conceptual understanding in mathematics without the supporting basic skills, and basic skills are weakened by a lack of understanding. The essential connection between basic skills and understanding of concepts in mathematics was perhaps most eloquently explained by U.C. Berkeley mathematician Hung-Hsi Wu in his paper, *Basic Skills Versus Conceptual Understanding: A Bogus Dichotomy in Mathematics Education*.[75]

The obstacles faced by parents opposed to the NCTM programs for their children were formidable. The events leading to the creation of the Princeton Charter school illustrate some of the generic difficulties.

In 1991 a group of about 250 parents of school children in Princeton, New Jersey petitioned the board of education for a more systematic and challenging math program. They found the one in use to be vague and weak. Many of the teachers did not even use textbooks. When parents asked about what was being taught in the classrooms, they were told that the curriculum was not very important, that "one size does not fit all," and, repeating the dictum of 1930s Progressivists, that the teachers were there to "teach children, not curricula." When parents complained of deficiencies in what little curriculum even existed, they were treated as if their cases were new and unrelated to other complaints. These responses have been reported by parents in many other school districts as well.

Test scores in Princeton were among the highest in the state, but that was not the result of a well designed academic program. Many highly educated parents, including Princeton University faculty, were providing tutoring and enrichment for their own children. Other children with limited resources in the Princeton Regional School system did not fare well in this highly progressivist environment.

Finding their requests ignored, the "Curriculumists," those parents favoring an organized coherent curriculum for all students, concentrated on winning school board seats. One of them, Chiara Nappi, a theoretical physicist at the Institute for Advanced Study in Princeton, won a seat in 1993. By 1994 the Curriculumists held a majority of positions on the school board. However, even with formal political power, the Curriculumists were unable to make substantive changes in the district. They eventually turned their attention to creating a charter school for grades K-8 whose focus would be the fundamental academic disciplines, and which would provide an atmosphere that affirmed academic achievement. However, even this effort was resisted by Progressivists in the district. Nevertheless, after considerable effort, the Princeton Charter School came into existence in 1997 and provided a genuine alternative to the educational philosophy of the school district.[76]

Parents in California were also alarmed by the mathematics programs their children were getting in school. California was ahead of the rest of the nation in implementing the approach to mathematics education envisioned in the *NCTM Standards* and *An Agenda for Action*. The 1985 *California Model Curriculum Standards, Grades Nine Through Twelve* already had prescriptions that closely resembled those in the *NCTM Standards* such as:

> The mathematics program must present to students problems that utilize acquired skills and require the use of problem-solving strategies. Examples of strategies that students should employ are: estimate, look for a pattern, write an equation, guess and test, work backward, draw a picture or diagram, make a list or table, use models, act out the problem, and solve a related but simpler problem. The use of calculators and computers should also be encouraged as an essential part of the problem-solving process. Students should be encouraged to devise their own plans and explore alternate approaches to problems.

As a consequence, mathematics reform along the lines of the weak *1989NCTM Standards* was well underway in California in the early 1990s. California was one of the first states to embrace the *NCTM Standards*, producing a state mathematics framework in 1992 that closely resembled the *NCTM Standards*. By 1994, the California State Board of Education had approved math curricula for grades K-8 aligned to the 1992 California mathematics framework, and by extension, the *NCTM Standards*.

The first significant parental rebellion in California occurred in Palo Alto, a highly educated community that included Stanford University faculty and business leaders. In May 1994, more than 600 parents signed a petition asking that the school district retain a traditional pre-algebra curriculum at one of the middle schools in the Palo Alto Unified School District. The district was about to replace the remaining traditional courses with a math program aligned to California's 1992 math framework. Finding the district uncooperative, 25 parents in Palo Alto formed "Honest and Open Logical Debate," or HOLD in February 1995, put up a website the next month, and within a short period of time there were nearly 500 households on the HOLD mailing list. The already considerable math credentials of HOLD members were increased by the support and participation of Henry Alder, a professor of mathematics at UC Davis, a former president of the Mathematical Association of America, and a former member of the California State Board of Education. Alder had long been advocating themes similar to those of HOLD.

HOLD criticized the 1992 California math framework and the *NCTM Standards*, and pointed to a decrease in Stanford Achievement Test scores coinciding with the implementation of "whole math" in district schools. From 1992 to 1994 the average overall student score for 8th grade math students had decreased from the 91st national percentile rank to the 81st. The decrease was more dramatic on the portion of the exam that tested computation. On that portion the scores dropped from the 86th percentile in 1992 to the 58th percentile in 1994. Parents took steps to compensate for the lack of computational skills taught to their children in school. According to Bill Evers, one of the cofounders of HOLD:

> Palo Alto School District parents are sufficiently discontented with the district's math performance that in massive numbers they are resorting to outside math tutoring programs. Forty-eight percent of parents report providing outside help in math for their children (in the middle schools, this number rises to 63 percent). The math-basics group HOLD's own informal survey of the best-known commercial math programs shows that Palo Alto parents are spending at least $1 million a year for math tutoring.[77]

With the extra tutoring, the district scores partially rebounded the following school year.

At the southern end of the state, four parents, Paul Clopton, Larry Gipson, Mike McKeown, and Martha Schwartz came together in the Autumn of 1995 to form "Mathematically Correct." Their common nemesis was fuzzy math and in particular, College Preparatory Mathematics (CPM), a secondary, integrated math program. Martha Schwartz had just participated in a group of parents that had collected more than 1,000 signatures for a petition to a school district in Torrance, California asking for a traditional alternative to CPM. This same program had been introduced in San Diego schools in 1993, and the founding parents found common cause in confronting the problems this curriculum and others like it were causing school children.

The founders of Mathematically Correct had credentials in science and mathematics that could not easily be dismissed. Gipson was a professional engineer; Clopton a statistician working for the Department of Veterans Affairs in San Diego; Schwartz was finishing up a Ph.D. in geophysics; McKeown was a faculty member at the Salk Institute for Biological Studies in San Diego (a few years later, McKeown accepted a professorship in the Division of Biology and Medicine at Brown University). They were soon joined by others, notably Wayne Bishop, a professor and former chair of the Mathematics Department at California State University, Los Angeles, and Frank Allan, a former president of the NCTM. Both had many years

of experience dealing with mathematics education issues, and both were critics of the *1989NCTM Standards*.

Organized for the explicit purpose of assisting parents dissatisfied with "fuzzy math" in their children's schools, Mathematically Correct attracted a large number of supporters (including the author of this chapter). Like other groups of its type, Mathematically Correct charged no dues, had no annual budget, and there was no formal membership. Mathematically Correct was fueled entirely by the energy and dedication of its supporters, especially its webmaster, Paul Clopton. In the decade of the 1990s, Mathematically Correct emerged as the most influential and effective organization to challenge the NCTM agenda. It served as a national clearing house for information and advice on K-12 mathematics education. Its supporters entered the political process, met with reporters and politicians, served on California government panels and commissions related to mathematics education, and testified before national boards and the U.S. Congress. Mathematically Correct and HOLD played important roles in establishing the California mathematics standards in 1997, a topic taken up in the next section.

Mathematically Correct also came into contact with other like-minded parent organizations, such as Parents Raising Educational Standards in Schools (PRESS), based in Milwaukee, Wisconsin; Concerned Parents of Reading, Massachusetts; Concerned Parents in Petaluma, California; Mountain View Achievement; Santa Monicans Working for Equity and Excellence in Public Schools; as well as many others. All of these grassroots parents' organizations were opposed to NCTM aligned math curricula in the schools and had information or websites linked from the Mathematically Correct website. The Internet was a powerful organizing tool for parents of school children during the 1990s.

A parent group in Plano, Texas took the unusual step of suing the school district in order to find an alternative to one of the NSF funded math programs. In 1996, Plano Independent School District (PISD) began piloting Connected Math in four of its nine middle schools. By the summer of 1998, some parents were objecting to the program. One parent who criticized Connected Math was removed from a textbook advisory committee in the Fall of 1998. Another parent was prevented from passing out information critical of Connected Math at PISD informational meetings, and was also prevented from collecting signatures to a petition asking for an open discussion with parents about the merits of the program. As a result, parents formed the organization, MathChoice, in January 1999. Frustrated that the district continued to ignore parental complaints about the program, MathChoice started another petition drive in May 1999. The petition was really just a one page form that parents could fill out requesting an alternative math class for their children. Each form began with the sentence, "This petition is for the addition of a specific, traditional/conventional academic class in the course of study of math for the parent or guardian's child named: ..." The district responded by sending letters to parents in the school district that countered the petition, effectively putting an end to the petition drive. However, by the end of May, 521 signatures had already been collected.

Finding their petitions ignored, the Plano parents turned to litigation. In October 1999, MathChoice incorporated as the Plano Parental Rights Council. They attained non-profit status from the IRS the following spring and elected Susan Sarhady as president. Seeking class certification, six parents filed suit in federal court "against the Plano Independent School District for violations of the parentís constitutionally protected rights of free speech/expression, equal protection and the fundamental right as parents to direct the education and upbringing of their children."[78] In May 2000, a federal judge ruled that "Plano Independent School District cannot be compelled to offer an alternative middle-school math program despite the objections of some parents to the new Connected Math approach...." However, the judge "also found that certain allegations by the parents should go forward to trial ... The lawsuit claim[ed] that the First Amendment rights of several parents were violated when they were prohibited from distributing or displaying materials opposing the Connected Math program at several meetings."[79]

Another important parents' organization emerged in 1999 in New York City. The New York City school system had been awarded an Urban Systemic Initiative grant from the NSF in 1994, and New York state had a Statewide Systemic Initiative grant. The New York Urban Systemic Initiative reported training 4,200 teachers in inquiry-based curricula, and more than 700 teachers in the use of calculators for high school mathematics courses. According to posted reports to the NSF, the USI also implemented "exemplary curricula" in over 5,000 classrooms in New York City.

New York City Schools are grouped into 32 community school districts. Each has its own school board and superintendent. Community School District 2 consists of about 42 schools serving 22,000 racially diverse students. The district included relatively affluent neighborhoods as well as neighborhoods with substantial concentrations of lower income families and recent immigrants.

Beginning in 1993, teachers were trained in materials created by Marilyn Burns, a prominent teacher trainer cited earlier in this section. From 1995 to 1998 pilot programs in TERC's Investigations in Number, Data and Space and Connected Mathematics gradually expanded in District 2. By 1999 TERC and CMP were used districtwide. The NSF funded curriculum, Mathematics: Modeling Our World (ARISE), was scaled up for use in all of the high schools in 2000 and 2001.

In May 1999, Elizabeth Carson, a concerned parent of a middle school student, began a search for allies to try to reverse the districtwide implementation of weak NCTM aligned mathematics programs. The result was an alliance consisting of parents, teachers, City University of New York mathematics professors, and a substantial portion of the faculty of the math department of the Courant Institute of New York University. They named themselves "New York City HOLD" (NYC HOLD) after the Palo Alto group. Allies of NYC HOLD communicated with each other largely through the Internet, but many of them met weekly at New York University for planning sessions or discussions with interested visitors, including education journalists. On June 6, 2001, NYC HOLD held an open forum for parents and teachers in an auditorium in the New York University Law school. Approximately 350 people attended, and plans were subsequently made for other projects to challenge the nearly exclusive use of NCTM-aligned curricula in the schools.

In the decade of the 1990s, the parent organizations in California, especially Mathematically Correct, experienced the greatest successes, not only in blocking the use of dubious classroom materials, but also in implementing coherent, effective mathematics policies at the state level. The California program at the end of the 20th century included high quality mathematics textbooks and a testing system aligned to the California standards. However, parent organizations did not accomplish these changes unilaterally. Many other sectors of society and prominent individuals played critical roles. They included classroom teachers and principals, university mathematicians, legislators, state school board members, journalists, and two successive governors.

Mathematicians, California, and the Nation

No state had so great a national impact as California on mathematics education during the 1990s. This was due in part to the fact that California was the most populous state, and as a consequence, the demands placed on textbook publishers to sell to the California market influenced what was available to the rest of the nation. But the effect of California's new educational policies during the middle and late 1990s went deeper. Perhaps the clearest indications of the importance of California's choices were the harsh public denunciations by both the NCTM and the NSF of California's 1997 mathematics standards immediately following their release. This will be contrasted with the strong support given by university mathematicians and parent groups, later in this section. By the end of the decade, it was clear that

California's mathematics program threatened a century of progressivist domination in K-12 mathematics education.

Not since the New Math period of the 1960s had university mathematicians played such important roles in K-12 education as in California during the 1990s. Mathematicians were involved in developing the state mathematics standards, the California mathematics framework, and in evaluating textbooks and professional development programs for teachers in California. Some mathematicians also helped to write and develop textbooks for the textbook adoption process of 2001. During the decade of the 1990s, at the national level, there were extended discussions about K-12 mathematics among research mathematicians through their professional meetings and magazines.[80] The result was a greater participation by university mathematicians nationally in matters related to mathematics education, including interactions with parent organizations.

In California, by the mid-1990s, the dramatic failures of "whole language learning" in teaching primary grade students to read had already cost the education establishment substantial credibility with the public. Analogous failures in mathematics education were opening opportunities for critics of constructivist education policies to make changes. Mathematicians and parent activists displaced, to a considerable degree, the education professionals and college of education faculty who would normally be entrusted to work out the policy details for K-12 mathematics education. As a consequence, mathematicians were naturally drawn into educational and political debates.

An early example was the participation of Professor Wayne Bishop on a Mathematics Task Force formed by the state Superintendent of Schools, Delaine Eastin, in 1995. The 25 member Task Force was charged with recommending ways to improve mathematics instruction in California. Bishop publicly resigned from the Task Force in order to make known his disagreement with the weak recommendations the Task Force was making.

Following the release of the Task Force report, Professor Henry Alder addressed the California State Board of Education in December 1995. He articulated the views of the emerging parent organizations in California and indirectly reinforced Bishop's symbolic resignation. Alder recommended "a revision or perhaps even a complete rewriting of the 1992 California Mathematics Framework rather than issue a supplement." Paving the way for broader participation in mathematics education policy making, Alder also recommended that a:

> new task force to be charged with your Board's directives be appointed in consultation with all affected constituencies, with an appropriate mix of expertise from all segments interested in and involved in mathematics education. This means, in particular, that its membership should not be dominated by those who prepared the 1992 California Mathematics Framework and those who constituted the Mathematics Task Force.[81]

The State Board of Education agreed with the critics and scheduled a rewrite of the 1992 Framework two years ahead of the normal time table. By this time there was considerable public pressure to improve the teaching of reading and mathematics in the schools. The legislature had just passed a bill that required school districts to include the teaching of basic skills in reading and math as part of their curriculum. Governor Wilson signed this "ABC Bill" in October 1995, and it became law in January 1996. The ABC laws had virtually no effect on school districts, which were generally run by committed constructivists, but political leaders felt compelled to do something about the mounting failures in education. Whole language and whole math--the math programs aligned to the NCTM Standards and 1992 California framework--were widely viewed as responsible for depriving children of fundamental skills.

At the national level during the mid-1990s much attention was focused on international comparisons of student mathematics achievement. The first available results of the Third International Mathematics and Science Study (TIMSS) were released in November 1996. U.S. 8th graders scored slightly below the international average in mathematics. The second TIMSS report comparing 4th grade students in math was released in June 1997. U.S. fourth grade students were slightly above the international average among the participating countries. The final report compared students at the end of high school and was released in 1998. The mathematics achievement of U.S. 12th graders was among the lowest of the participating nations. The TIMSS data contained valuable information, but it had relatively little political impact on the ensuing debates, as both sides cited the studies to reinforce their respective positions. However, TIMSS researchers did express support for the *NCTM Standards*. The eighth grade study found that[82]:

> **Ninety-five percent of U.S. teachers stated that they were either "very aware" or "somewhat aware" of current ideas about teaching and learning mathematics.** When asked to list titles of books they read to stay informed about current ideas, one third of U.S. teachers wrote down the names of two important documents by the National Council of Teachers of Mathematics, Curriculum and Evaluation Standards and Professional Teaching Standards [bold in original].
>
> U.S. teachers believe that their lessons are already implementing the reform recommendations, but the findings described so far in this chapter suggest that their lessons are not. When asked to evaluate to what degree the videotaped lesson was in accord with current ideas about teaching and learning mathematics, almost 75 percent of the teachers respond either "a lot" or "a fair amount." This discrepancy between teachers' beliefs and the TIMSS findings leads us to wonder how teachers themselves understand the key goals of the reform movement, and apply them in the classroom.

The report suggests, without any experimental support, that if the U.S. teachers had properly followed the constructivist NCTM Standards, then U.S. students would have performed better in the study. However, it is possible that the teachers were correct in asserting that they were following "current ideas about teaching and learning mathematics," and there was no "discrepancy." The report went on to say:

> Over 80 percent of the teachers in the study referred to something other than a focus on thinking, which is the central message of the mathematics reform movement. The majority of the teachers cited examples of hands-on math or cooperative learning, which are techniques included among the reform recommendations. However, these techniques can be used either with or without engaging students in real mathematical thinking. In fact, the videotape study observed many examples of these techniques being conducted in the absence of high-quality mathematical content.

The authors of the report did not consider the possibility that the NCTM reform movement itself was a contributing cause of poor student performance. Even a cursory examination of the NCTM aligned math curricula would show a disturbing lack of "high quality mathematical content." Nevertheless, the TIMSS report prescribed still more of the same reform:

> **These findings suggest that the instructional habits and attitudes of U.S. mathematics teachers are only beginning to change in the direction of implementation of mathematics reform recommendations. Teachers' implementation of the reform still concentrates on isolated techniques rather than the central message, which is to focus lessons on high-level mathematical thought.** The finding that almost 20 percent of the teachers believed that they had implemented this focus on mathematical thinking, despite experts' judgments that a high-

> quality sequence of mathematical ideas was virtually absent in their lessons, suggests that teachers may not yet understand what the reform movement means by this term [bold in original].

The growing criticisms of NCTM aligned reform curricula coming from professional mathematicians raised the possibility that the real focus of the reform movement was constructivist classroom techniques rather than "high-level mathematical thought." This possibility was not considered by the authors of the TIMSS reports.

Results from National Assessment of Educational Progress (NAEP) were released in February 1996. While the nation as a whole made some improvements, California's fourth graders scored below their peers in 40 states and came out ahead of only those in Mississippi. A careful analysis of NAEP trends for the nation as a whole was published by the Brookings Institution later in September 2000, but California's relative downward slide reinforced the political will toward writing explicit mathematics standards and rectifying the 1992 framework to include more attention to basic skills.[83] Adding to California's concerns was a steady increase in remedial math courses on the 23 campus California State University (CSU) system. The percentage of entering freshmen failing an entry level math test used by the CSU, and requiring remedial courses, steadily increased from 23% in 1989 to 54% in each of 1997 and 1998. While there was no proof that the decrease in math skills was caused by the constructivist math programs in the schools, school mathematics seemed to be getting worse rather than better as the NCTM reform agenda expanded.

In January 1997, a committee called the Academic Content and Performance Standards Commission (Standards Commission) was charged with writing mathematics (and other subject matter) standards for California and submitting its draft to the State Board of Education for final approval. The committee consisted of non expert citizens appointed through a political process. The majority of the Standards Commissioners were largely in agreement with the constructivist policies of the past. The result was a set of standards submitted to the Board in the Fall of 1997 that not only embraced the constructivist methods that California was trying to escape, but was also incoherent and full of mathematical errors.

Members of the State Board asked for help from Stanford University mathematics professors Gunnar Carlsson, Ralph Cohen, Steve Kerckhoff, and R. James Milgram. In a few short weeks they rewrote the standards, corrected more than 100 mathematical errors, and eliminated all pedagogical directives, leaving the standards pedagogically neutral. The final revisions, including those made by the State Board itself, resulted in a document that would allow teachers to use constructivist methods or direct instruction, or whatever classroom techniques worked for them, so long as they taught all of the grade level content standards. The mathematics framework was regarded as the proper document for discussions of pedagogy, but not the standards themselves. This was what the State Board was looking for, and the mathematicians' standards were adopted by California in December 1997. These standards were clear, coherent, and met the criteria set by the California legislature to be competitive with math standards of the highest performing countries.

Professor Hung-Hsi Wu did a careful analysis of the California standards, that the board adopted, in comparison to the draft submitted by the Standards Commission which the Board rejected. Wu found numerous mathematical errors and lack of clarity and cohesion in the rejected standards, in contrast to an overall soundness and clarity in the adopted standards.[84] In 1998, the Fordham Foundation conducted an independent review of the mathematics standards from 46 states and the District of Columbia, as well as Japan. California's new board approved mathematics standards received the highest score, outranking even those of Japan.[85]

The NCTM responded to the new California mathematics standards with denunciations. The cover story of the February 1998 News Bulletin of the NCTM began with:

> Mathematics education in California suffered a serious blow in December. Over protests from business, community, and education leaders, California's state board of education unanimously approved curriculum standards that emphasize basic skills and de-emphasize creative problem solving, procedural skills, and critical thinking.

NCTM President Gail Burrill used strong words in a letter to the president of the California Board of Education. She wrote, "Today's children cannot be prepared for tomorrow's increasingly technological world with yesterday's content...The vision of important school mathematics should not be one that bears no relation to reality, ignores technology, focuses on a limited set of procedures ...California's children deserve more."

The NSF also condemned California's deviation from constructivism. Luther Williams, the National Science Foundation's Assistant Director for Education and Human Resources, wrote a letter to the Board on NSF letterhead stationary. Williams' letter, dated December 11, 1997, explained that the Boardís decision to adopt the mathematics standards "vacates any serious commitment to elevating problem solving and critical thinking skills..." Williams added, "The Board action is, charitably, shortsighted and detrimental to the long-term mathematical literacy of children in California." Speaking for the National Science Foundation, he chastised, "We view the Board action in California with grave disappointment and as a lost opportunity for the cities we support?indeed, for the entire state."

The condemnations of the new California math standards by non mathematicians turned into an avalanche. Judy Codding, a vice president of the National Center on Education and the Economy (NCEE), had served on California's Standards Commission. She made no secret of her opposition to the new standards. Speaking at an NCEE conference, she declared "I will fight to see that California math standards are not implemented in the classroom." California Superintendent of Schools Delaine Eastin also denounced the math new standards written by the Stanford mathematicians as being "dumbed down." According to Eastin, the California standards represented "a decided shift toward less thinking and more rote memorization." Eastin also complained that with the new standards, "we're not even going to let [students] use a calculator before the sixth grade."[86] The statewide chairs of the Academic Senates of the UC, CSU, and California Community College systems, none of whom were mathematicians, also joined the chorus. They issued a joint statement condemning the adoption of California's math standards and falsely declared that "the consensus position of the mathematical community" was in opposition to the new standards, and generally in support of the rejected, standards, written by the Standards Commission.

California mathematicians put an end to the rumor that there was any consensus in the mathematics community against the new California standards. More than 100 mathematics professors from colleges and universities in California added their names to an open letter in support of the California mathematics standards. The signatories included chairs of the math departments at Caltech, Stanford, several UC and CSU campuses, as well as community college faculty. Jaime Escalante also added his name in support.[87] One of the flash points in the disagreement about the standards was whether long division should be taught in K-12 beyond the case of single digit divisors, and this was indicated in the open letter. A detailed explanation of the importance of the division algorithm by two mathematicians was later provided for the benefit of teachers.[88]

The criticisms of the California standards in the press diminished after a few months, and work proceeded on developing the *Mathematics Framework for California Schools*. R. James Milgram and Hung-Hsi Wu played fundamental roles in the many mathematical portions of the final document. Important contributions were also made by others, including cognitive psychologist David Geary of the University

of Missouri and educational researcher Douglas Carnine together with other members of the National Center to Improve the Tools of Educators (NCITE) at the University of Oregon. The State Board of Education contracted with NCITE to perform a study "to locate high quality research about achievement in mathematics, review that research, and synthesize the findings...From a total of 8,727 published studies of mathematics education in elementary and secondary schools, the research team identified 956 experimental studies. Of those, 110 were deemed high quality research because they met tests of minimal construct and internal and external validity."[89]

The *Framework* was adopted by the California State Board of Education in December 1998. A system was developed for textbook adoptions in California which included panels of mathematicians, as well as different panels whose membership consisted primarily of classroom teachers. The panels of mathematicians were charged with evaluating mathematics curricula submitted for statewide adoption, on the basis of the quality of mathematical content. This screening process by mathematicians contributed important voices to California's 1999 and 2001 textbook adoption process. Most of the panel members came from California universities, but not all. Richard Askey of the University of Wisconsin at Madison and Ralph Raimi of the University of Rochester participated on the 1999 panels.

Even after California identified textbooks aligned to its new state standards, resistance to the California standards at the local school district level was significant. Decisions at the district level were largely under the control of administrators who looked for guidance from the NCTM, the NSF, and sometimes the NCEE. The new content standards of California would not easily be accepted. In one case which received front page coverage in the Los Angeles Times, a critic of the California math standards threatened a hunger strike in order to increase the chances of classroom use of NCTM aligned math programs.[90] Nevertheless, as early as 1999 some school districts were coming to grips with the new guidelines. The Los Angeles Unified School District included Paul Clopton, Hung-Hsi Wu, Ze'ev Wurman, one of the co-founders of HOLD, and Barry Simon, the mathematics department chair at Caltech, on a textbook selection committee. While the recommendations of these highly knowledgeable participants were largely ignored, the mere fact of their participation was a departure from the past.

One of the signal events of 1999 was the release of Liping Ma's book, *Knowing and Teaching Elementary Mathematics*.[91] Ma compared answers to elementary school math questions by 23 U.S. elementary school teachers to those by 72 Chinese elementary school math teachers. Of the U.S. teachers, 12 were participating in an NSF sponsored program whose "goal was to prepare excellent classroom mathematics teachers to be inservice leaders in their own school districts or regions."[92] The remaining U.S. teachers were interns, each with one year experience teaching. The interns were to receive Masters Degrees at the end of the summer during which interviews took place. By contrast, most Chinese teachers had only 11 or 12 years of formal education, completing only the ninth grade in high school followed by two or three years of normal school. In spite of their fewer years of formal education, the Chinese teachers demonstrated much greater understanding of fundamental mathematics than did their U.S. counterparts. Ma masterfully explained the interrelationships of pedagogy and content at the elementary school level and drew important lessons from her investigations. Liping Ma's book was embraced by all sides in the math wars. That unique distinction offered at least some hope that the warring factions could at some point find substantive issues upon which to agree.

Other events in 1999 were less unifying. In October, the U.S. Department of Education released a list of ten recommended math programs, as indicated at the beginning of this chapter. The programs were designated as either "exemplary" or "promising," and those programs are listed in the appendix to this chapter. The *Open Letter to United States Secretary of Education Richard Riley* was published on November 18, 1999 as a full paid ad in the Washington Post, paid for by the Packard Humanities Institute. The authors of the letter were David Klein, Richard Askey, R. James Milgram, and Hung-Hsi

Wu. Descriptions of some of the shortcomings of the "exemplary" and "promising" curricula were later published in the American School Board Journal.[93] The NCTM responded to the open letter by explicitly endorsing all ten of the "exemplary" and "promising" programs (see appendix).

The ten "exemplary" and "promising" math programs were chosen by an "Expert Panel" designated by the U.S. Department of Education . The one mathematician on the Expert Panel, Manuel Berriozabal, publicly distanced himself from its decisions. The *Christian Science Monitor* reported that "Berriozabal abstained or voted against all 10 programs," and:

> "The panel was a good idea," Dr. Berriozabal says, "but we made some bad judgments. From the best I could tell, none of the programs we selected as 'promising' or 'exemplary' had any kind of long-term track record of achievement." After Berriozabal arrived in Washington, the panel began debating the criteria to determine a successful program. Berriozabal thought that long-term proof of achievement should top the list. Most others on the panel wanted to require programs to conform to NCTM standards--then gauge achievement.[94]

Not all mathematicians were in agreement with the Open Letter. The most prominent critic of the Open Letter was Hyman Bass, the incoming president of the American Mathematical Society. Bass posted a message on a national listserve that denounced the Open Letter.[95] The only program he defended in his message was Connected Math, though he did acknowledge that this grade 6-8 "exemplary" program did not include any treatment or explanation of division of fractions, as pointed out by Richard Askey. Bass complained that the Open Letter politicized the discussion. As reported in the *Notices of the American Mathematical Society*:

> Bass disagrees with many of the conclusions in the letter, but his main objection is that the letter has inserted the debate over mathematics curricula "into the world of journalism and politics, whereÖserious and balanced discussion will no longer be possible." He also expressed concern that "What appear to be very sensible reservations about what the Department of Education did [have] become in fact part of a veiled and systematic assault on the professional education community."[96]

In his email message, Bass expanded on his political objections:

> Mathematically Correct, an important agent in promoting this Open Letter, has been politically active around the country. In Massachusetts it is allied with efforts of the Deputy Commissioner of Education, Sandra Stotsky, to review proposed revisions to the State Framework. Her ideological and uninformed opposition to "constructivist ideas" has reached the incredible state where she is opposed to inclusion of discussion of "Classical Greek constructions" as being "constructivist pedagogy." Is this what serious mathematicians want to associate themselves with?

Formerly a research associate at Harvard and an expert on children's reading, Dr. Sandra Stotsky was one of a handful of education leaders at the state or national level who endorsed the Open Letter. Chester Finn, a former U.S Assistant Secretary of Education, and Lisa Graham Keegan, the Superintendent of Public Education of Arizona also endorsed the Open Letter to U.S. Secretary Riley. Bass' accusation that Stotsky was opposed to "Classical Greek constructions" in geometry was completely without basis, as she later informed him; Bass had unwittingly misinterpreted another person's sarcastic comments. Indeed, Stotsky was on record as wanting a strong set of high school geometry standards in the revision of the mathematics curriculum framework for Massachusetts and sought the advice of Harvard mathematics professor Wilfried Schmid. Schmid provided generous assistance in the development of the new mathematics framework for Massachusetts, which suffered from similar opposition as the one in California. The Massachusetts math framework, much like California's, deviated from the constructivist

prescriptions of the NCTM.[97] Schmid, who was critical of NCTM aligned curricula, also signed the Open Letter.[98]

Several months after the publication of the Open Letter to Secretary Riley, the U.S. Department of Education designated two more curricula as "promising": *I Can Learn* and *Growing With Mathematics*. The Department of Education praised these two programs, for their alignment to the *NCTM Standards*, among other reasons.

At the state level, California all but ignored the U.S. Department of Education recommendations. Of the 12 "exemplary" and "promising" math programs, only the UCSMP grade 7 and 8 textbooks were adopted in 1999 in California, and none were accepted for statewide adoption in 2001. Several NSF sponsored math curricular programs were submitted for statewide adoption in California in 1999 and 2001, but due to deficiencies in mathematical content, none were adopted in either year.

Given the size of the California textbook market, it is not surprising that there were heated debates between mathematicians, on the one hand, and the mathematics education community, on the other, about specific curricula and the influence of the California standards. As an illustration, the creators of one of the "exemplary" programs, Core-Plus, posted an article on their website from Western Michigan University that included as part of a rebuttal of criticisms of Core-Plus:

> ... Mr. Milgram also has a strong anti-reform agenda and was a leader in the campaign that led to the new California Mathematics Standards that have been widely criticized as retrograde by the mathematics education community.[99]

The culminating event for mathematics education of the 1990s occurred in April 2000 when the NCTM released a new document entitled, *Principles and Standards for School Mathematics*(PSSM).[100] PSSM was a revision of the *1989 NCTM Standards* intended to address some of the criticisms of the earlier document. The writing teams for the year 2000 national standards began work on the PSSM in 1997, and many organizations were solicited for suggestions. The PSSM is a 402 page document organized into eight chapters, and it is similar in many respects to its predecessor, the *1989 NCTM Standards*. Some of the more radical declarations from the *1989 NCTM Standards* were eliminated, and slightly greater emphasis was given to the importance of arithmetic algorithms and computational fluency in the new document. The PSSM provided guidelines for spans of grades: pre-kindergarten to grade 2, 3-5, 6-8, and 9-12. As explained by Ralph Raimi who served on a committee of the American Mathematical Society to make recommendations for the new standards, the revisions fell short of what many of the critics would have preferred:

> As Joan Ferrini-Mundy, its principal editor, explained in her September Notices [of the American Mathematical Society] article, NCTM this time commissioned the commentary of many mathematicians, including committees of AMS, MAA, and SIAM, upon an earlier draft prepared for us. I myself served on the AMS committee and (by commission) as an individual too. NCTM solicited public advice at large, and I know several who also attempted to link the mathematical world with the new document, but the effort was to little avail; the message--the "vision" of PSSM--remains, in my vision, much the same as that of the original 1989 Standards.
>
> PSSM continues to abhor direct instruction in, among other things, standard algorithms, Euclidean geometry, and the uses of memory. Though like its predecessor it has the word "standards" in its title, it is not a set of standards in the usual meaning of the term, for it refuses to say what exactly a child should learn in thirteen years of schooling. Long division? Quadratic formula? How to compute the quotient of two fractions? (See p. 218

> of PSSM for an enlightening discussion.) Proof of a theorem on inscribed angles? Trigonometric identities? PSSM will neither affirm or deny, lest it seem to dictate content.[101]

Concluding Remarks

At the end of the 20th century, mathematics education policies in U.S. public schools were in a state of flux. Disagreements between parents and mathematicians, on the one hand, and professional educators, on the other, continued without clear resolution. Wilfried Schmid described the disagreements at the end of the 1990s succinctly:

> The disagreement extends over the entire mathematics curriculum, kindergarten through high school. It runs right through the National Council of Teachers of Mathematics (NCTM), the professional organization of mathematics teachers. The new NCTM curriculum guidelines, presented with great fanfare on April 12 [2000], represent an earnest effort at finding common ground, but barely manage to paper-over the differences.
>
> Among teachers and mathematics educators, the avant-garde reformers are the most energetic, and their voices drown out those skeptical of extreme reforms. On the other side, among academic mathematicians and scientists who have reflected on these questions, a clear majority oppose the new trends in math education. The academics, mostly unfamiliar with education issues, have been reluctant to join the debate. But finally, some of them are speaking up.
>
> Parents, for the most part, have also been silent, trusting the experts--the teachers' organizations and math educators. Several reform curricula do not provide textbooks in the usual sense, and this deprives parents of one important source of information. Yet, also among parents, attitudes may be changing...
>
> The stakes are high in this argument. State curriculum frameworks need to be written, and these serve as basis for assessment tests; some of the reformers receive substantial educational research grants, consulting fees or textbook royalties. For now, the reformers have lost the battle in California. They are redoubling their efforts in Massachusetts, where the curriculum framework is being revised. The struggle is fierce, by academic standards.[102]

The stakes are high not only for mathematics education in the public schools, but also for the nation's colleges and universities. Through a domino effect that begins in the elementary school grades and works its way up the educational ladder, the so-called reforms promoted by the NCTM, and other education organizations, are sure to affect university level mathematics education. Without adequate foundations in arithmetic skills and concepts from elementary school, entering middle school students will be unable to progress to algebra. Without strong foundations in algebraic skills and ideas, the doors to subsequent meaningful mathematics courses will be closed. University mathematicians are worried. As Hung-Hsi Wu explained in 1997:

> This reform once again raises questions about the values of a mathematics education ...by redefining what constitutes mathematics and by advocating pedagogical practices based on opinions rather than research data of large-scale studies from cognitive psychology.

> The reform has the potential to change completely the undergraduate mathematics curriculum and to throttle the normal process of producing a competent corps of scientists, engineers, and mathematicians. In some institutions, this potential is already a reality.[103]

In an era of international competition, it is unlikely that the public will tolerate such trends indefinitely. It was the broad implementation of the NCTM reforms themselves that created the resistance to them. Ironically, the extraordinary success in disseminating progressivist mathematics programs may, in the long run, be the principal reason for the demise of progressivism in mathematics education.

Appendix

The letter below was written in response to the Open Letter sent to U.S. Education Secretary Richard Riley from more than 200 mathematicians and prominent individuals. That Open Letter was published on November 18, 1999 in the Washington Post. It called for the withdrawal of the U.S. Department of Education's recommendations of the following mathematics programs, labeled by the Education Department as "exemplary" or "promising":

Exemplary

Cognitive Tutor Algebra

College Preparatory Mathematics (CPM)

Connected Mathematics Program (CMP)

Core-Plus Mathematics Project

Interactive Mathematics Program (IMP)

Promising

Everyday Mathematics

MathLand

Middle-school Mathematics through Applications Project (MMAP)

Number Power

The University of Chicago School Mathematics Project (UCSMP)

November 30, 1999

Secretary Richard W. Riley

United States Secretary of Education

400 Maryland Avenue

Washington, DC 20202

Dear Mr. Secretary:

In light of the recent paid advertisement in the Washington Post requesting that you withdraw the list of exemplary and promising mathematics programs, the Board of Directors of the National Council of Teachers of Mathematics wishes to inform you of their unconditional support for the work of the Expert Panel, the criteria used by the Panel, the process employed by the Panel, and the quality and appropriateness of their final recommendations.

We are deeply disappointed that so many eminent and well-intentioned mathematicians and scientists have chosen to attack the work of the Panel. We note, however, that the advertisement represents the opinion of a small, but vocal, minority of mathematicians and scientists, many of whom have little direct knowledge of the elementary and secondary school mathematics curriculum nor how to make it responsive to the needs of all students.

Unfortunately, while NCTM is working diligently and successfully to engage mathematicians and mathematics teachers at all levels in the process of setting high standards for school mathematics, the authors of the Post advertisement seem determined unilaterally to undermine the programs that the Expert Panel has found to be exemplary and promising. We believe that the Panel took a hard look at quality, alignment with sound standards, and most importantly, how the various programs affect student learning. The ten programs recommended by the Expert Panel have already had a positive influence on thousands of young people. Thanks to work of the Panel, these programs can be expected to have an equally positive impact on millions of young people in the coming years. For reasons that we do not understand, this fact appears to seriously bother many of the individuals who allowed their names to be associated with the Post ad.

Mr. Secretary, NCTM's Board of Directors believes that the Department has performed a great service by providing this list of programs. We thank you and your colleagues for supporting the work of the Expert Panel and look forward to continuing to work with you on behalf of the mathematics education of our nation's youth.

Sincerely,

John A. Thorpe

Executive Director

Acknowledgements. The author would like to express his gratitude to Henry Alder, Richard Askey, Wayne Bishop, Williamson Evers, R. James Milgram, Chiara Nappi, Ralph Raimi, Diane Ravitch, Sandra Stotsky, and Edie Pistolesi for their helpful comments and suggestions on draft versions of this chapter

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