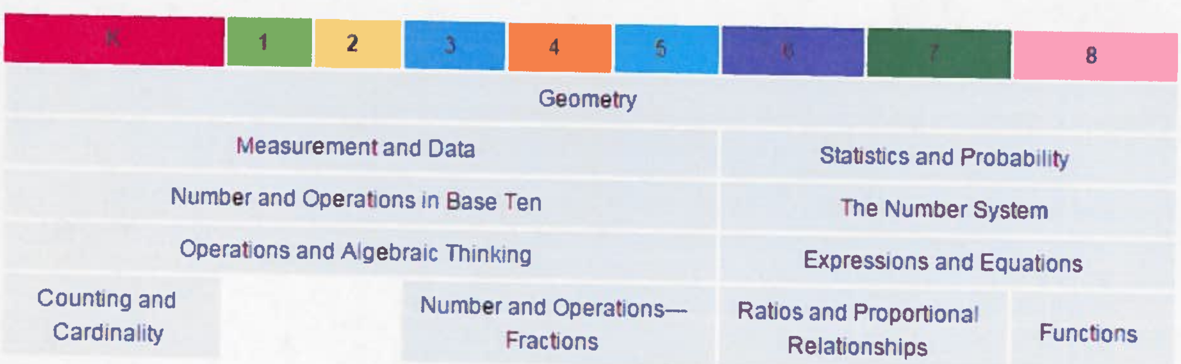
## Structure of Arkansas Math Frameworks Grades K - 8 (2004 – 2012)

### Mathematics Curriculum Framework

| Standards | |
|---|---|
| **Number and Operations** | |
| 1. Number Sense | Students shall understand numbers, ways of representing numbers, relationships among numbers and number systems. |
| 2. Properties of Number Operations | Students shall understand meanings of operations and how they relate to one another. |
| 3. Numerical Operations and Estimation | Students shall compute fluently and make reasonable estimates. |
| **Algebra** | |
| 4. Patterns, Relations and Functions | Students shall recognize, describe and develop patterns, relations and functions. |
| 5. Algebraic Representations | Students shall represent and analyze mathematical situations and structures using algebraic symbols. |
| 6. Algebraic Models | Students shall develop and apply mathematical models to represent and understand quantitative relationships. |
| 7. Analysis of Change | Students shall analyze change in various contexts. |
| **Geometry** | |
| 8. Geometric Properties | Students shall analyze characteristics and properties of 2 and 3 dimensional geometric shapes and develop mathematical arguments about geometric relationships. |
| 9. Transformation of Shapes | Students shall apply transformations and the use of symmetry to analyze mathematical situations. |
| 10. Coordinate Geometry | Students shall specify locations and describe spatial relationships using coordinate geometry and other representational systems. |
| 11. Visualization and Geometric Models | Students shall use visualization, spatial reasoning and geometric modeling. |
| **Measurement** | |
| 12. Physical Attributes | Students shall use attributes of measurement to describe and compare mathematical and real-world objects. |
| 13. Systems of Measurement | Students shall identify and use units, systems and processes of measurement. |
| **Data Analysis and Probability** | |
| 14. Data Representation | Students shall formulate questions that can be addressed with data and collect, organize and display relevant data t answer them. |
| 15. Data Analysis | Students shall select and use appropriate statistical methods to analyze data. |
| 16. Inferences and Predictions | Students shall develop and evaluate inferences and predictions that are based on data. |
| 17. Probability | Students shall understand and apply basic concepts of probability. |

*Each grade level continues to address earlier Student Learner Expectations as needed and as they apply to more difficult text.

## Structure of Common Core State Standards Grades K – 8



# Focus Areas By Grade Level Provided For Under Common Core

**In Kindergarten,** instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

1. Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

2. Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes

**In Grade 1**, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

1. Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

2. Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

3. Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.

4. Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry

**In Grade 2**, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

1. Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

2. Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

3. Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

4. Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

**In Grade 3**, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

1. Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

2. Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

4. Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

**In Grade 4**, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

1. Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

2. Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

3. Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

**In Grade 5**, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

1. Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

2. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

3. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

**In Grade 6**, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

1. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

2. Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4. Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability.

   Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected. Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

**In Grade 7**, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

4. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

**In Grade 8**, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality ($m$) is the slope, and the graphs are lines through the origin. They understand that the slope ($m$) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

   Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

# Key Shifts in Mathematics

## Introduction

The Common Core State Standards for Mathematics build on the best of existing standards and reflect the skills and knowledge students will need to succeed in college, career, and life. Understanding how the standards differ from previous standards—and the necessary shifts they call for—is essential to implementing them.

The following are the key shifts called for by the Common Core:

1. Greater **focus** on fewer topics

   The Common Core calls for greater focus in mathematics. Rather than racing to cover many topics in a mile-wide, inch-deep curriculum, the standards ask math teachers to significantly narrow and deepen the way time and energy are spent in the classroom. This means focusing deeply on the major work of each grade as follows:

- In grades K–2: Concepts, skills, and problem solving related to addition and subtraction
- In grades 3–5: Concepts, skills, and problem solving related to multiplication and division of whole numbers and understanding fractions
- In grade 6: Ratios and proportional relationships, and early algebraic expressions and equations
- In grade 7: Ratios and proportional relationships, and arithmetic of rational numbers
- In grade 8: Linear algebra and linear functions

This focus will help students gain strong foundations, including a solid understanding of concepts, a high degree of procedural skill and fluency, and the ability to apply the math they know to solve problems inside and outside the classroom.

# The Standards for Mathematical Practice

## 1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## 2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## 4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## 5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

**6. Attend to precision.**
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**7. Look for and make use of structure.**
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 - 3(x - y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

**8. Look for and express regularity in repeated reasoning.**
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y - 2)/(x - 1) = 3. Noticing the regularity in the way terms cancel when expanding (x - 1)(x + 1), (x - 1)(x2 + x + 1), and (x - 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content**
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

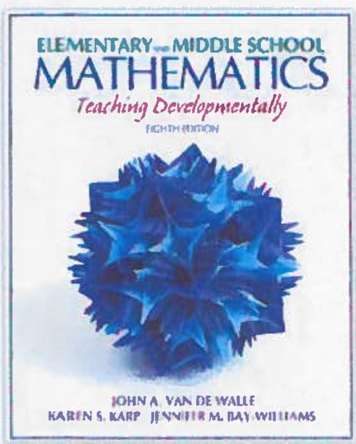
In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

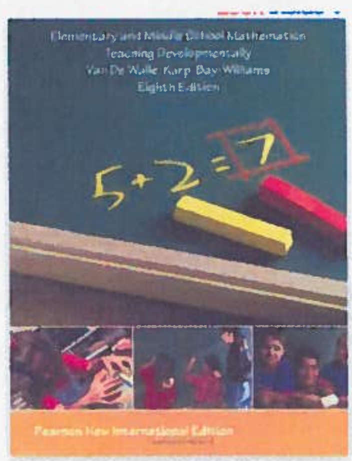
Carol Dweck: http://www.mindsetworks.com/webnav/dr-dweck-interviews.aspx

Jo Boaler: http://www.youcubed.org/

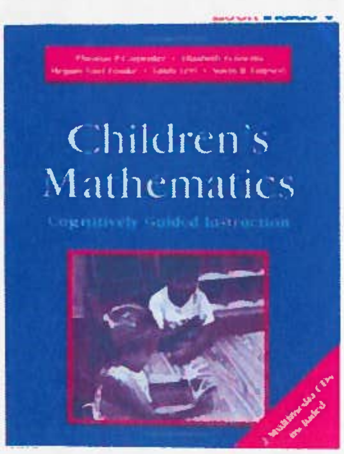
Karen Karp: http://amte.net/conferences/conf2013/featkarp

http://amte.net/group/research/09-13/dr-karen-karp




Linda Levi: http://www.promisingpractices.net/program.asp?programid=114



# LEARNING MATHEMATICS IN AN

# INQUIRY-BASED CLASSROOM

## What Parents Can Do to Help their Child with Mathematics

**Listen to your child's thinking and ask questions**

- Why do you think that?
- Can you explain how you got that?
- How do you know?
- Does your answer make sense?
- Can you solve it a different way?

**Practice/reinforce strategies used at school**

- Try not to tell your child a strategy. The strategy will come with understanding and practice. If your child is stuck, try to help them make sense of the problem by asking questions.
- Ask your child word problems as they come up in everyday life.
- Always ask "WHY?"

## Helping your Child Learn Mathematics

*Booklet created by the US Department of Education*

**Introduction**

What kind of attitude do you have toward math? Do you believe that math skills are important job and life skills? Do you see math as useful in everyday life? Or do you dread doing things that involve math – figuring out how much new carpet you'll need, balancing the checkbook, reading the technical manual that came with the DVD player? How you answer these questions indicates how you may be influencing your child's attitudes toward math – and how he approaches learning math.

Although parents can be a positive force in helping children learn math, they also can undermine their children's math ability and attitudes by saying things such as: "Math is hard," or "I'm not surprised you don't do well in math. I didn't like math either when I was in school," or "I wasn't very good in math and I'm a success, so don't worry about doing well." Although you can't **make** your child like math, you can encourage her to do so and you can take steps to ensure that she learns to appreciate its value in both her everyday life and in preparing for her future. You might point out to her how fortunate she is to have the opportunity to learn mathematics today – when mathematics knowledge can open the door to so many interesting and exciting possibilities.

In everyday interactions with children, there are many things parents can do – and do without lecturing or applying pressure – to help children learn to solve problems, to communicate mathematically and to do demonstrate reasoning abilities. These skills are fundamental to learning mathematics.

Let's look closely at what it means to be a problem solver, to communicate mathematically and to demonstrate mathematical reasoning ability.

**A problem solver** is someone who questions, finds, investigates and explores solutions to problems; demonstrates the ability to stick with a problem to find a solution; understands that there may be different ways to arrive at an answer; and applies math successfully to everyday situations. You can encourage your child to be a good problem solver by including him in routine activities that involve math – for example, measuring, weighing, figuring costs and comparing prices of things he wants to buy.

**To communicate mathematically** means to use mathematical language, numbers, charts or symbols to explain things and to explain the reasoning for solving a problem in a certain way, rather than just giving the answer. It also means careful listening to understand others' ways of thinking and reasoning. You can help your child learn to communicate mathematically by asking her to explain what she must do to solve a math problem or how she arrived at her answer. You could ask your child to draw a picture to show how she arrived at the answer.

**Mathematical reasoning ability** means thinking logically, being able to see similarities and differences in objects or problems, making choices based on those differences and thinking about relationships among things. You can encourage your child's mathematical reasoning ability by talking frequently with him about these thought processes.

**Some Important Things Your Child Needs to Know About Mathematics**

You can help your child learn math by offering her insights into how to approach math. She will develop more confidence in her math ability if she understands the following points:

1. **Problems Can Be Solved in Different Ways**
   Although most math problems have only one answer, there may be many ways to get to that answer. Learning math is more than finding the correct answer; it's also a process of solving problems and applying what you've learned to new problems.
2. **Wrong Answers Can Be Useful**
   Accuracy is always important in math. However, sometimes you can use a wrong answer to help your child figure out why she made a mistake. Analyzing wrong answers can help your child to understand the concepts underlying the problem and to learn to apply reasoning skills to arrive at the correct answer. Ask your child to explain how she solved a math problem. Her explanation might help you discover if she needs helps with number skills, such as addition, subtraction, multiplication and division or with the concepts involved in solving the problem
3. **Take Risks!**
   Help your child to be a risk taker. Help him see the value of trying to solve a problem, even if it's difficult. Give your child time to explore different approaches to solving a difficult problem. As he works, encourage him to talk about what he is thinking. This will help him to strengthen math skills and to become an independent thinker and problem solver.
4. **Being Able to Do Mathematics in Your Head Is Important**
   Mathematics isn't restricted to pencil and paper activities. Doing math "in your head" (mental math) is a valuable skill that comes in handy as we make quick calculations of costs in stores, restaurants or gas stations. Let your child know that by using mental math, her math skills will become stronger.

# Administrator's Guide to Frequently-Asked Questions

# About a CGI/ECM Classroom

- **Why change the way math is taught?**
  - To many people, math is a series of unconnected ideas and formulas that don't make sense.
  - Many children and adults have the opinion that they aren't good at math and readily tell people they "hate math."
  - The traditional instructional model focused on procedural understanding of how to solve specific types of problems as opposed to conceptual understanding of how the mathematics within those problems and within the solutions works.
  - Overall, in the traditional instructional model, the procedures taught to solve specific math problems have not resulted in children being able to apply that knowledge to problems presented in a slightly different way. Also, this procedural knowledge is not effectively carried from grade level to grade level and much time is spent every year in re-teaching procedures from previous years.

- **Why is my child drawing pictures or using a roundabout way to get to the answer of a math problem? Why can't they just use the standard algorithm?**
  - Educational research shows that students must go through a cycle of concretely and/or pictorially solving a problem before they can make sense of a more abstract problem solving process such as using an algorithm.
  - Drawing pictures or using seemingly ineffective strategies (like using a number line, breaking numbers apart, or counting) helps students to initially make sense of the problem.
    - This process allows students to build their understanding of how mathematics works
  - Teachers will eventually move students to solving problems using the algorithms we are all used to using, but it is important for those algorithms to make sense to the student or that knowledge is easily lost or corrupted.
    - Many times, after students have moved to solving a problem using an algorithm, when they are presented with a similar problem presented in a different context or one that involves slightly different mathematics, students will revert to drawing pictures or using other strategies to solve the problem. This is a good thing! It gives the student an entry point into the problem solving process that they might not ordinarily have if they had just learned the algorithm from the beginning.

- **Why don't teachers just tell students how to do the math?**
  - We want students to build their own understanding of math.
  - When a teacher tells a student how to solve a problem, the student is trying to build upon the teacher's understanding of the problem instead of their own.
    - The teacher's understanding may or may not make sense to the student.
    - Every year we see that this method does not result in learning that "sticks."
      - Many students remember the procedure just long enough to pass the test.
    - Students must make sense of the problem in order to be an effective problem solver in the future.
  - Students only learn the mathematical connections the teacher tells them about instead of constructing their own.
    - Teachers will tell you that students make mathematical connections their teachers never even thought about when asked to think about the mathematics on their own.

- **My child has always gotten A's in math and now he/she has a C. Why is he/she struggling so much?**
    - Many students are not used to learning in this way and we are asking them to think differently about mathematics.
    - Problem solving is much different than being asked to solve specific math problems. Problem solving is a skill that requires perseverance and grit. Although this is challenging for many students in the beginning, in the long run, it is a skill that will serve them well in the future.
    - Rote memorization of facts and procedures is easy for some students. Conceptual understanding can initially be more difficult, but will make their math experiences as they move to Algebra and Geometry much easier in the end.

- **Why doesn't my child have a math textbook?**
    - We want students to build upon understandings of previously learned concepts, to make sense of problems, and to use what they know to build a strategy to solve the problem on their own (even though, initially that strategy may be rather unsophisticated). The format of typical math textbooks undermines this process.
    - Math textbooks usually show students how to do the mathematics, then give examples of several problems and their solutions, have "naked number" practice problems, followed by a few word problems.
        - In an inquiry-based classroom, we present problems in context first because the context allows students to make sense of the problem and gives them an entry point to solving the problem that might not occur in "naked number" problems.

- **Why don't I see much homework coming home?**
    - Homework and practice problems are not assigned to students until the teacher believes the student has a firm understanding of the mathematical concept. Teachers and students may work on a concept for several weeks in class before students are ever asked to practice in a homework assignment.
        - In the past we have assigned homework and practice hoping that conceptual understanding would develop out of practice. That conjecture has been proven wrong with most students.
        - Students who did not understand the procedure would practice that procedure incorrectly over and over again on their homework problems. Undoing that incorrect understanding is very difficult, if not next to impossible, for some students.
        - Practice is still important, but is used to build fluency and efficiency of problem solving strategies; not conceptual understanding.

- **How can I help my child at home?**
    - Don't show your children how to solve problems.
    - Help your child to make sense of the problem.
    - Allow your child to draw pictures or diagrams to help him/her make sense of the problem and how to solve it.
    - Listen to your child's thinking about how to solve problems and ask questions like:
        - Why do you think that?
        - Can you explain how you got that?
        - How do you know?
        - Does your answer make sense?
        - Can you solve it a different way?
        - Will the way you solved this problem always work?
    - Ask your child to solve problems that come up in your everyday lives.

- **How do you know this way of teaching math will work? Is anyone else using this method?**
    - There is a lot of research in the field of mathematics education to support the successful learning of mathematics in an inquiry-based classroom. Some examples are:
        - Carpenter, T.P., E. Fennema, M.L. Franke, L. Levi, S. Empson. 1999. Children's Mathematics; *Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.
        - Clements, C.H. 2004. Major themes and recommendation. In *Engaging young children in mathematics: Standards for early childhood mathematics education,* eds. D.H. Clements, J. Sarama, & A.M. DiBlase, 7 – 72. Mahwah, NJ: Erlbaum.
        - Hiebert, J., T.P. Carpenter, E. Fennema, K.C. Fuson, D. Wearne, H. Murray, A. Olivier, & P. Human. 1997. *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.
    - As our math specialists, coaches, and teachers attend workshops and conferences across the country, they collaborate with other specialists, coaches, and teachers who are experiencing major gains in student understanding of mathematics in inquiry-based classrooms.
    - Many districts across Arkansas, especially in Northwest Arkansas, have implemented inquiry-based learning in their mathematics classrooms.
    - Our teachers are seeing that their students intuitively understand many mathematical concepts. This intuition has been "taught out" of our students in the past by focusing on procedures. By focusing on these intuitive ideas that students already possess, our Bentonville teachers are seeing amazing mathematical understandings emerge from our students and these understandings are transferring from concept to concept and grade to grade.

# Arkansas Frameworks

Strand: Number and Operations

Standard 3: Numerical Operations and Estimation

Students shall compute fluently and make reasonable estimates

THE GOAL FOR EACH STUDENT IS PROFICIENCY IN ALL REQUIREMENTS AT CURRENT AND PREVIOUS GRADES

| | Kindergarten | Grade 1 | Grade 2 | Grade 3 | Grade 4 |
|---|---|---|---|---|---|
| Computational Fluency-Addition and Subtraction | NO.3.K.1<br>Develop *strategies* for basic addition facts<br>• counting all<br>• counting on<br>• one more, two more<br><br>NO.3.K.2<br>Develop *strategies* for basic subtraction facts<br>• counting back<br>• one less, two less | NO.3.1.1<br>Develop *strategies* for basic addition facts<br>• counting all<br>• counting on<br>• one more, two more<br>• doubles<br>• doubles plus one or minus one<br>• make ten<br>• using ten frames<br>• *Identity Property* (add zero)<br><br>NO.3.1.2<br>Develop *strategies* for basic subtraction facts<br>• relating to addition<br>Ex.<br>Think of 7 – 3 = ___ as 3 + ___ = 7<br>• one less, two less<br>• all but one<br>Ex.<br>9 – 8, 6 – 5<br>• using ten frames of the answers | NO.3.2.1<br>Develop *strategies* for basic addition facts<br>• counting all<br>• counting on<br>• one more, two more<br>• doubles<br>• doubles plus one or minus one<br>• make ten<br>• using ten frames<br>• *Identity Property* (add zero)<br><br>NO.3.2.2<br>Demonstrate multiple *strategies* for adding or subtracting two-*digit whole numbers*<br>• *Compatible Numbers*<br>• *compensatory numbers*<br>• informal use of *commutative* and *associative properties of addition* | | |

Comparison of Addition and Subtraction Standards

Strand: Number and Operations
Standard 3: Numerical Operations and Estimation
Students shall compute fluently and make reasonable estimates

| THE GOAL FOR EACH STUDENT IS PROFICIENCY IN ALL REQUIREMENTS AT CURRENT AND PREVIOUS GRADES | | | | | |
|---|---|---|---|---|---|
| | Kindergarten | Grade 1 | Grade 2 | Grade 3 | Grade 4 |
| Computational Fluency-Addition and Subtraction | | | NO.3.2.3<br>Demonstrate *computational fluency* (accuracy, efficiency and flexibility) in addition facts with addends through 9 and corresponding subtractions<br>Ex.<br>9+9=18 and 18-9=9<br>add and subtract *multiples* of ten | NO.3.3.1<br>Develop, with and without appropriate *technology*, *computational fluency*, in multi-*digit* addition and subtraction through 999 using contextual problems<br>• *strategies* for adding and subtracting numbers<br>• *estimation* of sums and *differences* in appropriate situations<br>• relationships between operations | NO.3.4.1<br>Demonstrate, with and without appropriate *technology*, *computational fluency* in multi-*digit* addition and subtraction in *contextual problems* |

Comparison of Addition and Subtraction Standards

| | Common Core State Standards | | | | | |
|---|---|---|---|---|---|---|
| **Operations and Algebraic Thinking:** Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. | **Kindergarten** | **First Grade** | **Second Grade** | **Numbers and Operations in Base Ten:** Use place value understanding and properties of operations to perform multi-digit arithmetic. | **Third Grade** | **Fourth Grade** |
| | K.OA.1: Represent addition and subtraction with objects, fingers, mental images, drawings2, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. | 1.OA.1: Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. | 2.OA.1: Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. | | 3.NBT.2: Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. | 4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm |
| | K.OA.2: Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. | 1.OA.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. | 2.OA.2: Fluently add and subtract within 20 using mental strategies.2 By end of Grade 2, know from memory all sums of two one-digit numbers. | | | |
| | K.OA.3: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., | 1.OA.3: Apply properties of operations as strategies to add and subtract.3 Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of | 2.OA.3: Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even | | | |

Comparison of Addition and Subtraction Standards

| | | | | | | |
|---|---|---|---|---|---|---|
| **Operations and Algebraic Thinking:** Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. | 5 = 2 + 3 and 5 = 4 + 1). | addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.) | number as a sum of two equal addends. | **Numbers and Operations in Base Ten:** Use place value understanding and properties of operations to perform multi-digit arithmetic. | | |
| | K.OA.4: For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. | 1.OA.4 & 5: Understand subtraction as an unknown-addend problem. Relate counting to addition and subtraction. | 2.OA.4: Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. | | | |
| | K.OA.5: Fluently add and subtract within 5. | 1.OA.6: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). | | | | |