Journal for Research in Mathematics Education
1993, Vol. 24, No. 5, 428–441

MODELS OF PROBLEM SOLVING: A STUDY OF KINDERGARTEN CHILDREN'S PROBLEM-SOLVING PROCESSES

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Seventy kindergarten children who had spent the year solving a variety of basic word problems were individually interviewed as they solved addition, subtraction, multiplication, division, multistep, and nonroutine word problems. Thirty-two children used a valid strategy for all nine problems and 44 correctly answered seven or more problems. Only 5 children were not able to answer any problems correctly. The results suggest that children can solve a wide range of problems, including problems involving multiplication and division situations, much earlier than generally has been presumed. With only a few exceptions, children's strategies could be characterized as representing or modeling the action or relationships described in the problems. The conception of problem solving as modeling could provide a unifying framework for thinking about problem solving in the primary grades. Modeling offers a parsimonious and coherent way of thinking about children's mathematical problem solving that is relatively straightforward and is accessible to teachers and students alike.

The construction of a model or representation of a problem situation is one of the most fundamental problem-solving processes. Many problems can be solved by representing directly the critical features of the problem situation with an equation, a computer program, or a physical or pictorial representation. Modeling also turns out to be a relatively natural problem-solving process for young children. There is an extensive body of research documenting that even before they receive formal instruction in arithmetic, young children can solve a variety of different types of addition and subtraction word problems by directly modeling with counters the different action and relationships described in the problems (Carpenter, 1985; Fuson, 1992).

On the other hand, some of the most compelling exhibitions of problem-solving deficiencies in older students appear to have occurred because the students did not attend to what appear to be obvious features of problem situations. For example, in one frequently cited item from the third mathe-

The research reported in this article was supported in part by a grant from the National Science Foundation (MDR-8954679). The opinions expressed in this paper do not necessarily reflect the position, policy, or endorsement of the National Science Foundation.

matics assessment of the National Assessment of Educational Progress (1983), students were asked to find the number of buses required to transport 1128 soldiers if 36 soldiers could ride in each bus. Although almost three fourths of the 13-year-olds tested recognized that they needed to divide to solve the problem, only about a third of them rounded the quotient to the next largest whole number to account for the fact that the answer must be a whole number of buses. This and a number of similar examples suggest that many students abandon a fundamentally sound and powerful general problem-solving approach for the mechanical application of arithmetic and algebraic skills.

It appears that if older children would simply apply some of the intuitive, analytic modeling skills exhibited by young children to analyze problem situations, they would avoid some of their most glaring problem-solving errors. A fundamental issue would seem to be how to help children build upon and extend the intuitive modeling skills that they apply to basic problems as young children.

This study addresses this issue only indirectly. The subjects for the study were kindergarten children who had been in classes in which they had opportunities to model and solve problems; however, the focus of the study was not on the instruction but on the problem-solving processes of children. The study explores the potential for instruction to build upon and extend young children's problem-solving processes, but it is not a study of classroom instruction, and it does not address exactly how instruction should be designed to accomplish this task. The thesis of the study is that young children's problem-solving abilities have been seriously underestimated. As early as kindergarten, children are capable of solving a much wider variety of problems than previous research and the scope and sequence of the mathematics curriculum would suggest.

The specific purpose of the study was to investigate the problem-solving processes of kindergarten children who had spent a year in kindergarten classes in which they had the opportunity to explore a range of problem situations. We were particularly concerned with how an analytic framework based on the notion of problem solving as modeling explained the children's strategies for solving problems.

BACKGROUND

Children's solutions of basic addition and subtraction problems have been thoroughly documented (for reviews see Carpenter, 1985; Fuson, 1992). Although there is some variability in children's performance depending on the nature of the action or relationships in different problems, by the first grade most children can solve a variety of problems by directly modeling the action or relationships described in them. There are two accounts of the cognitive mechanisms involved in these solutions that differ in fundamental ways.[1] Riley, Greeno, and Heller (1983; Riley & Greeno, 1988) propose that children's ability to solve simple addition and subtraction problems depends on the availability of specific problem schemata for understanding the various semantic relationships in the problems. Briars and Larkin (1984), on the other hand, propose an analysis that, at the most basic level, does not include separate schemata for representing different classes of problems. Problems are mapped directly into the action schemata required to solve the problem. In other words, Riley and her associates hypothesize that specific knowledge about additive structures is required to solve basic addition and subtraction problems, whereas Briars and Larkin propose that children's initial solutions can be accounted for essentially in terms of the actions required to model the action in the problem.

Although the analysis proposed by Riley and her associates and that of Briars and Larkin were based on different assumptions about the cognitive mechanisms underlying children's performance, they drew on the same data sets, and they describe the same general levels of performance for addition and subtraction problems. It might be hypothesized, however, that if the analyses were extended to multiplication and division, some divergence might appear. Briars and Larkin's basic analysis extends directly to multiplication and division problems with relatively little further elaboration. Although some additional action schemata might be required to represent multiplication and division situations, most of the essential mechanisms should be in place. The analysis proposed by Riley and associates, however, requires a new set of problem schemata for understanding the structure of multiplication and division problems.

Problem Structure and Problem Difficulty

Most conceptual analyses of the structure of multiplication and division problems portray them as being much more complex than addition and subtraction problems (Greer, 1992; Schwartz, 1988; Vergnaud, 1983). The addition and subtraction problems given to young children generally involve only extensive quantities, quantities that can be represented directly. Multiplication and division problems, by their very nature, involve both extensive and intensive quantities, quantities that are derived from other quantities as in miles per hour or cookies per box. Thus, problem schemata for multiplication and division problems presumably would have to be more complex than those required for addition and subtraction problems.

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[1]There are other accounts of the processes involved in solving word problems, such as the linguistic analysis of children's difficulty in translating natural language statements into action on and relations among sets (e.g., Cummins, 1991; Cummins, Kintsch, Reusser, & Weimer, 1988). These accounts generally build on the basic semantic analyses and must ultimately deal with the issue of whether or not it is necessary to hypothesize specific knowledge of additive structures in accounting for children's behavior.

Results of studies of children's solutions of word problems involving addition, subtraction, multiplication, and division support the hypothesis that basic multiplication and division problems are more difficult than basic addition and subtraction problems. Kouba (1987, 1989), for example, reported that basic multiplication and division word problems were correctly solved by about 30% of the first-grade students she tested and about 70% of the third-grade students. These figures are approximately 20 to 30 percentage points below performance on simple addition and subtraction word problems (Carpenter, 1985; Kouba, 1987). The strategies that children used for multiplication and division word problems were consistent with the strategies that they use for addition and subtraction problems in that the children directly modeled the explicit action and relationships described in the problems. However, they seemed to have more difficulty modeling multiplication and division situations.

A fundamental question is whether the difference in difficulty reflects the fact that multiplication and division problems are inherently more difficult to solve than addition and subtraction problems or whether the performance differences are due in large part to differences in exposure. Additive situations may occur more frequently than multiplicative situations in a young child's environment. Furthermore, in most mathematics textbook series, simple addition and subtraction topics are introduced as early as kindergarten, but there is no substantive work with multiplication and division until the second grade. Studies of addition and subtraction have shown that giving children experience with addition and subtraction problem types that are not typically a part of the primary mathematics curriculum can significantly improve performance and reduce the discrepancy between problems that are considered relatively easy and certain problems that generally are considered more difficult (Carpenter, 1985; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Thus, there is a question of whether children as young as kindergarten age can solve multiplication and division problems given appropriate exposure or whether the problems are simply beyond their abilities.

METHOD

Subjects

The subjects for the study consisted of 70 children in six kindergarten classes in two schools. All children in the six classes who returned parent permission forms were included in the study. The six classes were taught by four teachers; two teachers taught separate morning and afternoon classes, and two taught all-day kindergarten classes.

Both schools served diverse populations. One school was located in a predominately upper-middle-class neighborhood, but it drew about a third of its population from a nearby low-income housing project. About 72% of the students who participated in the study were white, 12% African American, 9% Hispanic, and 7% Asian or Pacific Islander. About 31% of the students in the school participated in free or reduced-cost lunch programs. The second school served an economically mixed neighborhood. About 77% of the students in the study were white, 18% African American, and 5% Hispanic. About 27% of the students in the school participated in free or reduced-cost lunch programs.

Teacher Background

The purpose of providing background information about the teachers and about classroom instruction is to provide some context for the study, not to make claims about the consequences of a particular in-service program or about the effects of specific classroom instruction. The four teachers participated in a year-long in-service program called *Cognitively Guided Instruction* (Carpenter et al., 1989; Carpenter, Fennema, & Franke, 1992) that focused on children's informal or invented problem-solving strategies. Teachers discussed taxonomies for classifying different types of addition, subtraction, multiplication, and division word problems and identified the strategies that children tend to use to solve specific problems. The theme that tied together the characterization of children's problem-solving processes was that children initially tend to solve different problems by directly modeling the action or relationships described in the problems. Teachers discussed how they might use this information about children's problem solving in planning for instruction, but no specific guidelines for instruction or instructional materials were provided. The workshop included teachers from kindergarten through the third grade, and teachers were told that not all problems would be appropriate at all grade levels. The examples of children solving problems that were shared with the teachers in the workshops were all of children in the first and second grade. Because most of the available evidence suggested that most children begin to successfully solve the more difficult problem types during the first or second grade, kindergarten teachers were not encouraged to include all the types of problems discussed. They were to decide which problems would be appropriate for their students, but during the course of the year, they did discuss among themselves the problems that children in their respective classes were able to solve and the strategies they used to solve them.

Classroom Instruction

Information about instruction in the six classrooms is based on classroom observations and teacher reports. Three of the teachers were observed on seven or more occasions and one was observed four times. The teachers also were asked to describe the types of problems that they included in instruction throughout the year and how they were used. The solution of word problems played a prominent role in mathematics instruction in all six kindergarten classes; in fact, a large part of the instruction in mathematics was organized around the solution of word problems. Throughout the year children solved a variety of different problems. The teachers generally presented the problems and provided the children with counters that they could use to solve the problems, but the teachers typically did not show the children how to solve a particular problem. Children regularly shared their strategies for solving a given problem with the class or a small group, so a child might have learned a particular strategy by watching other children use it.

There was some variability in the problems given in the different classes, but by the time the children were interviewed for this study, children in all six classes had some experience with all but the two most difficult addition and subtraction problems and with grouping and partitioning (multiplication and division) problems. Most of the problem-solving experiences involved one-step word problems, but one teacher used some two-step problems and problems with extraneous information. There was some attempt to adapt numbers in the problems to the counting skills of individual children, but numbers up to 20 were used by the end of the year for most of the children. A few children worked with numbers up to 100.

Interview

Children were interviewed in May, when they had completed almost eight months of kindergarten. Children were interviewed by three trained interviewers. Each interviewer had observed in the kindergarten classes on at least four occasions. Each child was interviewed individually in a room apart from the classroom.

Each child was asked to solve the nine problems listed in Table 1. Each problem was read to the child by the interviewer. The interviewer reread the problem as many times as the child wished. If a child asked for specific information from the problem, the interviewer reread the entire sentence containing the information. Counters and paper and pencil were available on the table, and the children were told that they could use any of those materials to help them solve the problems.

The entire interview was audiotaped. Interviewers also coded children's responses as they solved each problem. If the interviewer could not understand what a child had done, she asked the child to explain what he or she had done. For each problem, there are a small number of easily identifiable strategies that children tend to use that provided the primary categories for the coding. The specific strategies used for each problem are described in the results section below. If a strategy did not conform to one of these characterizations, the interviewer made detailed notes about what the child had done and referred to the tapes for a verbatim account of what the child said.

Table 1
Interview Problems

Problem type	Order given	Problem
Addition and subtraction		
Separate (result unknown)	1	Paco had 13 cookies. He ate 6 of them. How many cookies does Paco have left?
Join (change unknown)	3	Carla has 7 dollars. How many more dollars does she have to earn so that she will have 11 dollars to buy a puppy?
Compare	5	James has 12 balloons. Amy has 7 balloons. How many more balloons does James have than Amy?
Multiplication and division		
Multiplication	2	Robin has 3 packages of gum. There are 6 pieces of gum in each package. How many pieces of gum does Robin have altogether?
Measurement division	4	Tad had 15 guppies. He put 3 guppies in each jar. How many jars did Tad put guppies in?
Partitive division	6	Mr. Gomez had 20 cupcakes. He put the cupcakes into 4 boxes so that there were the same number of cupcakes in each box. How many cupcakes did Mr. Gomez put in each box?
Multistep and nonroutine		
Division with remainder	9	19 children are going to the circus. 5 children can ride in each car. How many cars will be needed to get all 19 children to the circus?
Multistep	8	Maggie had 3 packages of cupcakes. There were 4 cupcakes in each package. She ate 5 cupcakes. How many are left?
Nonroutine	7	19 children are taking a mini-bus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has 7 seats. How many children will have to sit three to a seat, and how many can sit two to a seat?

Responses were coded both in terms of the strategy used and whether the answer was correct. Only valid strategies were coded in the strategies categories, but solutions that resulted in an incorrect answer due to a counting error were included. For a child's response to be coded as a valid strategy, the child had to use a strategy that would result in a correct answer if there were no counting error. Any strategy that would result in a correct answer if completed without error was coded as a valid strategy, whether or not the child modeled the action in the problem. However, the child had to complete the solution to the problem and report an answer that was off by no more than one or two because of a counting error to have the response coded as a valid strategy. If a child started using an appropriate strategy but could not complete the problem, that response was not coded as a valid strategy. Thus, the valid strategy category includes responses of children who correctly derived the correct answer plus responses of children who used a valid strategy but made a simple counting error.

A response was classified as being a *direct modeling* strategy if the child used counters or tally marks to model directly the action or relationships described in the problem. Simply using counters, tally marks, or other materials was not sufficient. If a child used counters or marks in a way that did not directly represent the specific action or relationships in the problem, the response was coded as *other.* A response was coded as a *counting* strategy when a child did not use counters, fingers, tally marks, or other materials to model directly the action in the problem but counted up or back from a given number or skip counted to calculate the answer. Responses were coded as *derived facts* when a child used recalled number facts to figure out the answer to a problem. In every case in which a child used a number fact, the fact that the child could recall was not the number fact given in the problem, so the child had to derive the answer. A response was coded as *uncodable* if a child got the correct answer but the interviewer could not reliably code the response on the basis of the child's actions and explanations. The specific strategies used to define these categories for each problem are described below.

RESULTS

Thirty-two children (46% of the total) used a valid strategy for all nine problems, although 13 of them made a minor counting error on one or two problems. Forty-four children (63% of the total) used a valid strategy and correctly calculated the answer to seven or more problems. Only 5 children got no problems correct. Results for each of the nine problems are summarized in Table 2. Most of the children who did not use a valid strategy for a given problem did not complete the problem using any identifiable strategy. Consequently, the incorrect responses provided few insights and are not included in the following analysis.

Table 2
Number of Children Correctly Solving Each Problem and the Number and Kind of Valid Strategies Used (N = *70)*

			Strategy				
Problem	Number correct	Valid strategies	Direct modeling	Counting	Derived fact	Other	Uncodable
Separate (result unknown)	51	62	54	5	2	0	1
Join (change unknown)	52	56	39	12	1	1	3
Compare	47	50	34	7	3	6	0
Multiplication	50	60	46	14	0	0	0
Measurement division	50	51	50	1	0	0	0
Partitive division	49	49	39	1	1	6	2
Division (remainder)	45	45	42	1	2	0	0
Multistep	45	47	44	0	3	0	0
Nonroutine	36	41	40	1	0	0	0

Separate (Result Unknown)

A total of 62 children used a valid strategy for the Separate (Result Unknown) problem. Fifty-four children directly modeled the action in the problem by making a set of 13 counters and removing 6 of them. Five children counted back from 13, keeping track of the number of counts on their fingers, with counters, or with tallies; and 2 children used a derived fact. For example, for one of the derived facts the child said that she knew that 3 and 10 was 13, so 13 take away 3 was 10, and take away 3 more was 7.

Join (Change Unknown)

Fifty-six children used a valid strategy for the Join (Change Unknown) problem. Thirty-nine children directly modeled by making a set of 7 counters and adding on counters until there was a total of 11. Twelve children counted up from 7 to 11. The derived fact was based on knowledge that 7 plus 3 make 10, 10 and 1 more make 11, and 3 plus 1 is 4. One child used a valid strategy that did not model the action in the problem, making a set of 11 counters and a set of 7 counters and lining them up in one-to-one correspondence. This strategy corresponds to the relationships described in a Compare problem.

Compare

Fifty children used a valid strategy for the Compare problem. Thirty-four directly modeled the problem, constructing the two distinct sets described in the problem. Twenty-four of these 34 children lined the two sets up in one-to-one correspondence. Ten children did not explicitly line the two sets up; instead they removed the number of counters in the smaller set from the larger set. Of the 7 children who used a counting strategy, 6 counted up and 1 counted back. Six children used valid strategies that did not model the action in the problem. Five of them made a set of 12 counters and removed 7, and the sixth made a set of 7 and added on counters up to 12. These strategies correspond to the strategies that model the action in Separate (Result Unknown) and Join (Change Unknown) problems, respectively.

Multiplication

Sixty children used a valid strategy for the Multiplication problem. Forty-six modeled the problem by making three sets with six counters in each set. Fourteen children used some form of counting without counters. Four of these children counted from one (1, 2, 3, 4, 5, 6 [pause], 7, 8, 9, 10, 11, 12 [pause], 13, 14, 15, 16, 17, 18). These children seemed to replace the sets of counters with three counting strings of six elements each. Three children counted starting with 6 (6 [pause], 7, 8, 9, 10, 11, 12 [pause], 13, 14, 15, 16, 17, 18), and 7 children knew that 6 and 6 was 12 and then counted on from 12 by ones.

Measurement Division

Fifty-one children used a valid strategy for the Measurement Division problem. Fifty children directly modeled the problem. Forty-two of them first counted out a collection of 15 counters, put them in sets of 3, and counted the number of sets. Nine additional children also made five sets of 3 counters each, but they did not initially count out the 15 counters. Instead, they kept track of the total number of counters used as they constructed their sets. When they had counted out a total of 15 counters in groups of three, they counted the number of sets. The child who used a counting strategy counted by threes.

Partitive Division

Forty-nine children used a valid strategy for the Partitive Division problem. Thirty-nine directly modeled the problem, making four sets with the same number of counters in each set and counting the number of counters in each set to find the answer. There were two basic variations of this strategy. Thirteen children systematically dealt the 20 counters one by one into four groups. Twenty-six children made four groups of counters and adjusted the number in each group until the groups each contained the same number of counters and all the counters were used up. Six of these 26 children started out making four groups of 4 counters each and then added a counter to each group. The other 20 children used a variety of trial-and-error strategies to equalize the groups. The child who used a counting strategy also used trial and error to eventually construct four counting strings that each contained 5 elements. The derived-fact strategy was based on the knowledge that 5 and 5 is 10 and 10 and 10 is 20. Of the 6 children who were coded as using other valid strategies, 4 used a strategy that would correspond to a Measurement Division problem. They made sets with 4 counters in each set and counted the number of sets. The other two children essentially made two groups of 10 and partitioned them in half.

Division with Remainder

Forty-five children used an appropriate strategy for the division problem in which they had to take into account that a whole car was needed to take care of the extra children. The problem is a Measurement Division problem, and 42 children modeled it as such. They constructed three sets with 5 counters in each and a final set of 4 counters. The counting strategy involved skip counting by 5, and the derived facts were based on knowledge of adding 5s and 10s.

Multistep

Forty-seven children used a valid strategy for the Multistep problem. Forty-four used counters or tallies to model the problem. Thirty-three of them made three groups of 4 counters each and then removed 5 of the counters one at a time. Eleven children made the three groups of counters and then removed one of the entire groups without recounting the individual counters in the group. They then removed one additional counter. Three children used number facts. They said that one of the groups was given away, so there were two groups left. They knew that 4 and 4 was 8 and they had to take away one more, so the answer was 7.

Nonroutine

For the problem in which 19 children had to be divided up 2 or 3 to a seat in a bus, responses were coded as correct if the children identified the number of seats that were occupied by two children and three children or if they identified the number of children who rode two to a seat and the number who rode three to a seat. Forty-one children used a valid strategy. Forty children used counters to model the problem. Seven of them designated the seven seats with a counter or location and systematically dealt the 19 counters out into the seven groups. The other 34 children used trial and error to place the counters in seven groups containing either 2 or 3 counters. This problem has some features that are similar to a Partitive Division problem, and these solutions are similar to the modeling strategies used for the Partitive Division problem. The child who used a counting strategy counted to 14 by twos and recognized that it would take 5 more to get to 19, so there would be five seats in which children would have to ride 3 to a seat.

DISCUSSION

Overall the kindergarten children in this study demonstrated remarkable success in solving word problems. Almost half the children used a valid strategy for all of the problems administered, and almost two thirds correctly solved seven or more of the problems. Almost 90% used a valid strategy for the most basic subtraction and multiplication problems, and over half the children were successful on even the most difficult problem.

The kindergarten children in this study were more successful in solving multiplication and division word problems than the first-grade students in Kouba's (1989) study, and they were about as successful as the third-grade students, although Kouba's third-grade students as a whole used more abstract strategies in their solutions. It also is interesting to compare the results for the problem in which children had to determine the number of cars needed to take 19 children to the circus with the results of the related National Assessment item described in the introduction to this article. Although it is unreasonable to presume that the problems are comparable, because there is a substantial difference in the numbers in the two problems, they do have the same general structure. For the National Assessment item, most of the 13-year-old students had difficulty deciding how to deal with the remainder, whereas the remainder caused no problem for most of the kindergarten children in this study. In fact, almost as many children correctly solved this problem as solved the Measurement Division problem that had no remainder.

This was not an experimental study, and it is not possible to attribute the success of the children in this study to the instruction they received or to any other specific factor. The results of this study do suggest, however, that children can solve a wide range of problems, including problems involving multiplication and division situations, much earlier than generally has been presumed. American textbooks typically include a narrow range of addition and subtraction problems in the primary grades (Stigler, Fuson, Ham, & Kim, 1986), and multiplication and division problems are not introduced until late in the second grade. The results of this study suggest that much more challenging problems involving a range of operations can be introduced early in the primary grades.

Problem Solving as Modeling

The kindergarten children's success in solving a range of problems can be accounted for in terms of what appears to be a unified approach to solving problems. With only a few exceptions, children's strategies could be characterized as directly representing or modeling the action or relationships described in the problems. Previous studies have shown that by the first and second grade children directly model the action in addition and subtraction word problems (Carpenter, 1985) and multiplication and division word problems (Kouba, 1989). This study demonstrates that children's ability to solve problems by modeling both extends to a broader range of problems and can be developed at a younger age than has been documented by prior studies.

We cannot claim that all kindergarten children would solve these problems in the same way that the children in this study solved them. Instruction did encourage the use of direct modeling to solve problems, and it is possible that if the instruction had a different focus the strategies might have looked different. However, the children in this study did model problems that differed from the problems they saw in class. At the very least, this study provides an existence proof that many kindergarten children can learn to solve problems by directly modeling the action and relationships in the problem, and they can apply this ability to a reasonably broad range of problems.

This study does not resolve the issue of what specific knowledge is required for solving particular types of problems. Children in this study were about as successful in solving multiplication and division problems as they were in solving addition and subtraction problems. These findings are consistent with the more general analysis proposed by Briars and Larkin (1984), but they do not conclusively demonstrate that specific multiplication and division schemata are not required for successfully solving multiplication and division problems as hypothesized by Riley et al. (1983). The results do suggest, however, that if specific multiplication and division schemata are required, these schemata are sufficiently well developed in many kindergarten children that they can solve multiplication and division problems by representing the action and relationships in the problems.

Our analysis indicates that at least at one level of detail it is not necessary to hypothesize specific addition, subtraction, multiplication, or division schemata in order to account for children's performance. Perhaps at a more fine-grained level of analysis specific schemata are necessary, but describing performance in terms of modeling offers a parsimonious and coherent way of thinking about children's mathematical problem solving that is relatively straightforward and is accessible to teachers and students alike (Carpenter, Fennema, & Franke, 1992).

The conception of problem solving as modeling could provide a unifying framework for thinking about problem solving in the primary grades. Although we cannot make any specific claims for the effects of instruction, it is worth noting that this is the conception that the teachers of the kindergarten students in this study shared. There is more to problem solving than modeling, but modeling seems to be a basic process that comes relatively naturally to most primary grade children. If we could help children to build upon and extend the intuitive modeling skills that they apply to basic problems as young children, we would have accomplished a great deal by way of developing problem-solving abilities in children in the primary grades.

Furthermore, modeling provides a framework in which problem solving becomes a sense-making activity. As a consequence, a focus on problem solving as modeling may do more than just provide children with cognitive schemes for solving problems. It seems likely that it will also have an impact on children's conception of problem solving and themselves as problem solvers. If from an early age children are taught to approach problem solving as an effort to make sense out of problem situations, they may come to believe that learning and doing mathematics involves the solution of problems in ways that always make sense.

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